The Bayesian Infinitesimal Jackknife for Variance

Tamara Broderick
Associate Professor,
MIT

With: Ryan Giordano 

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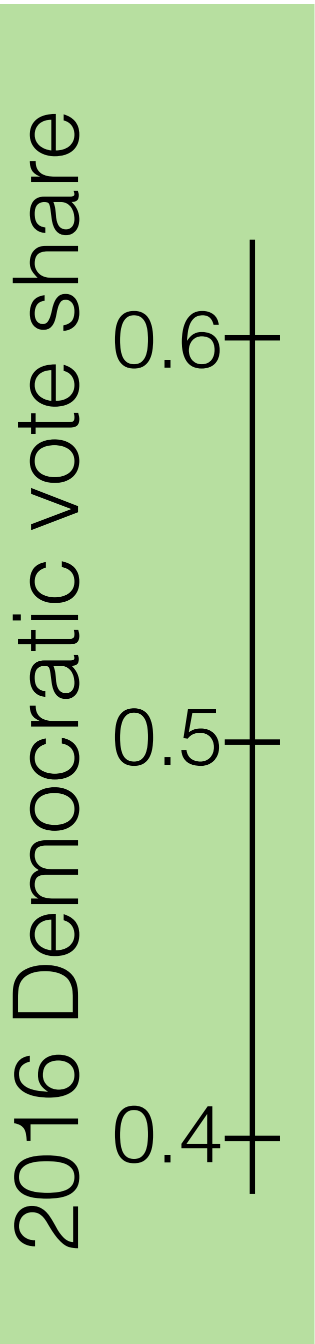

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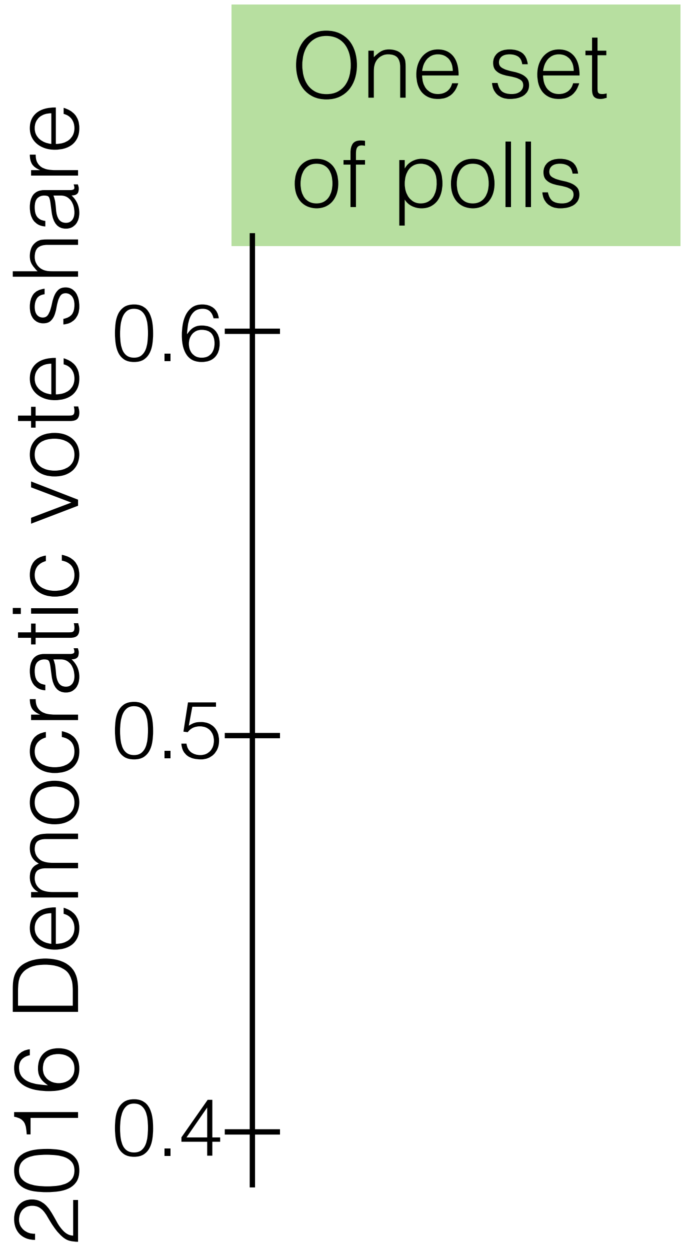

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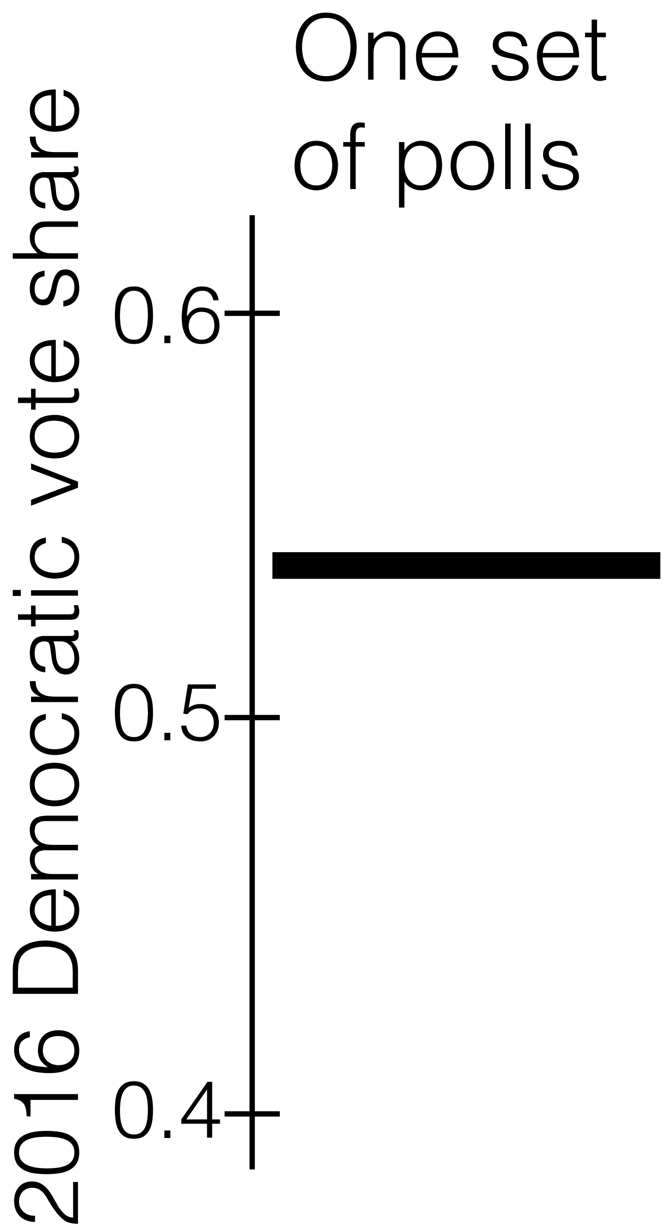

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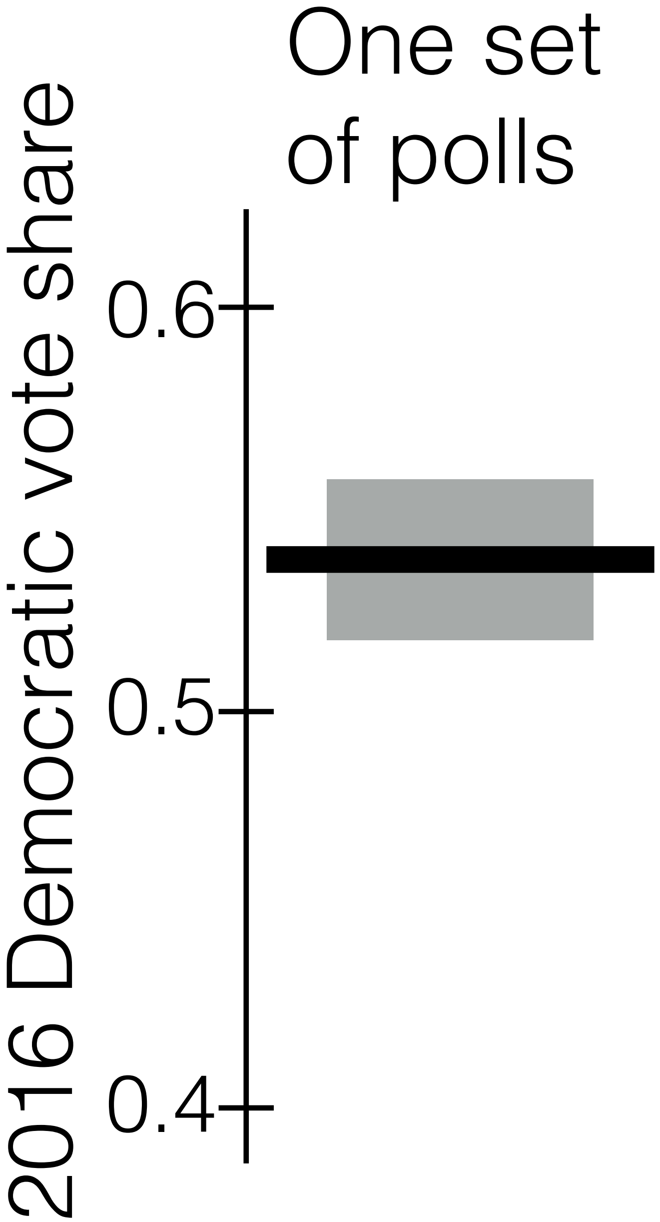

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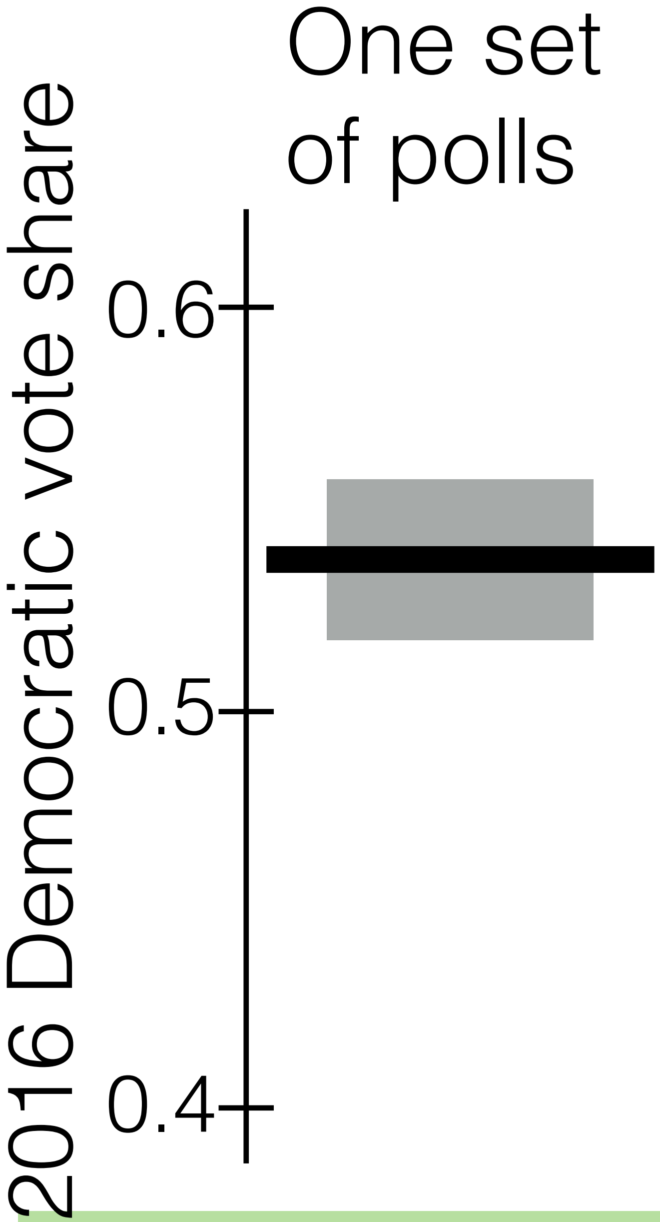

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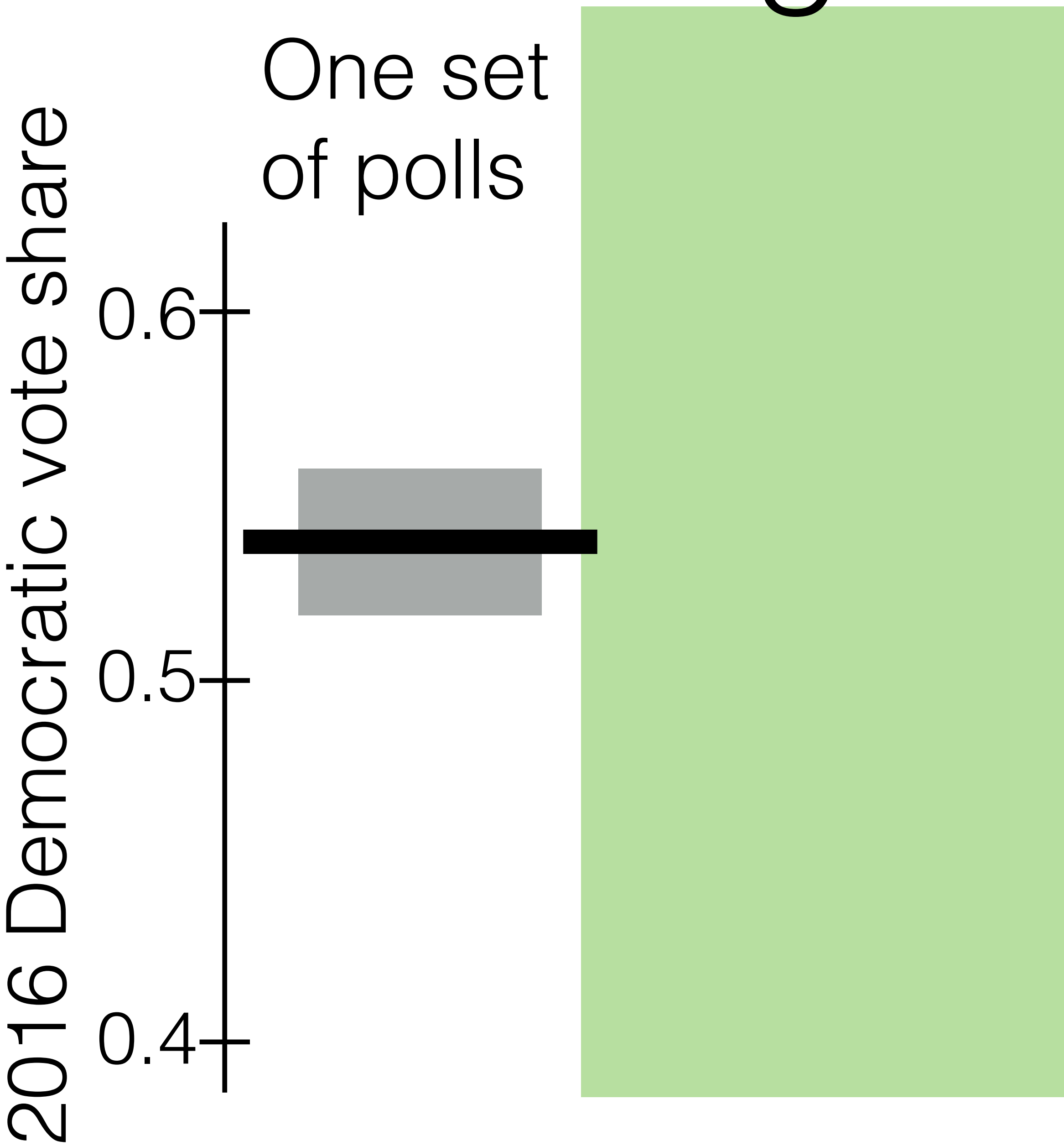

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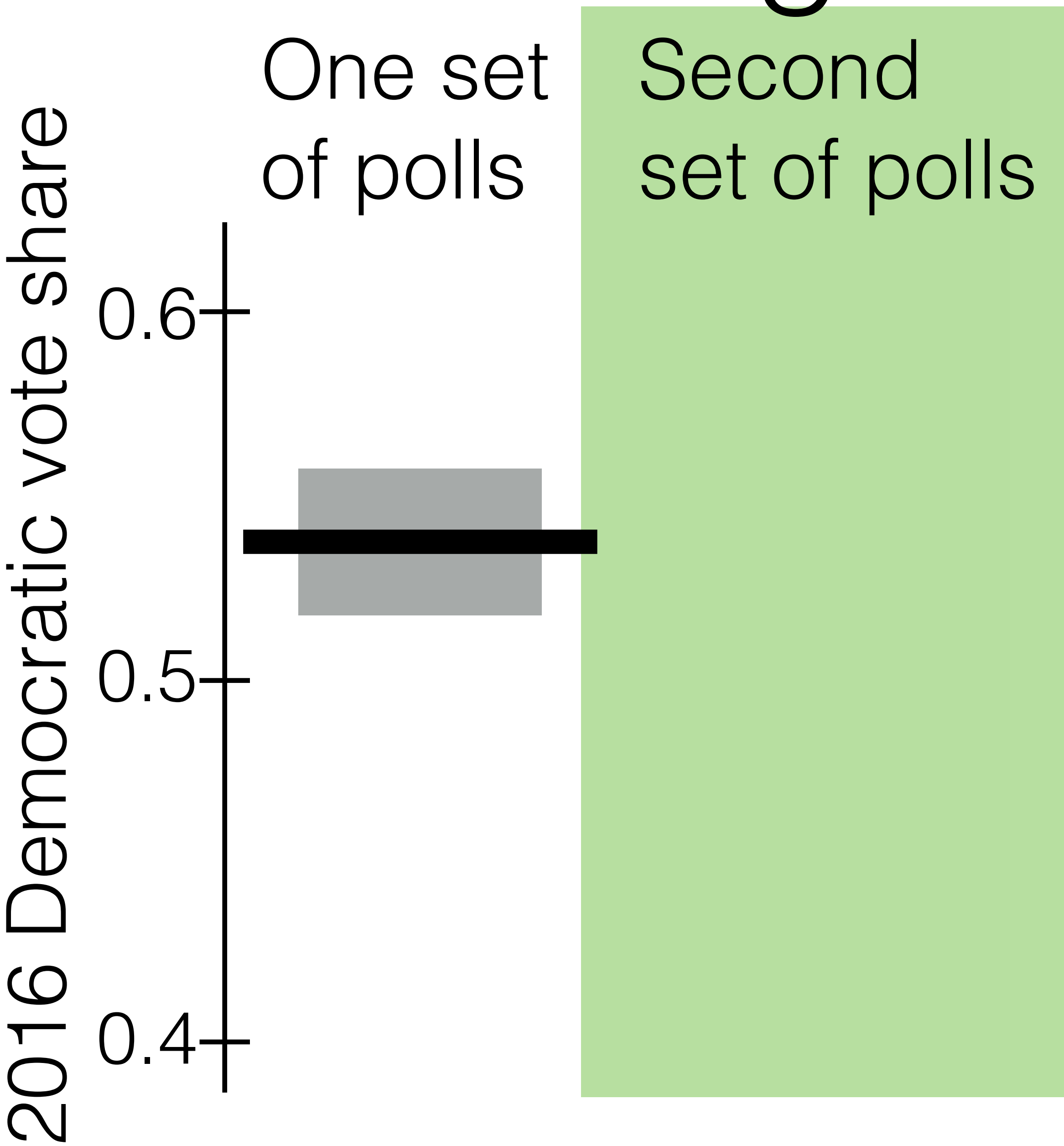

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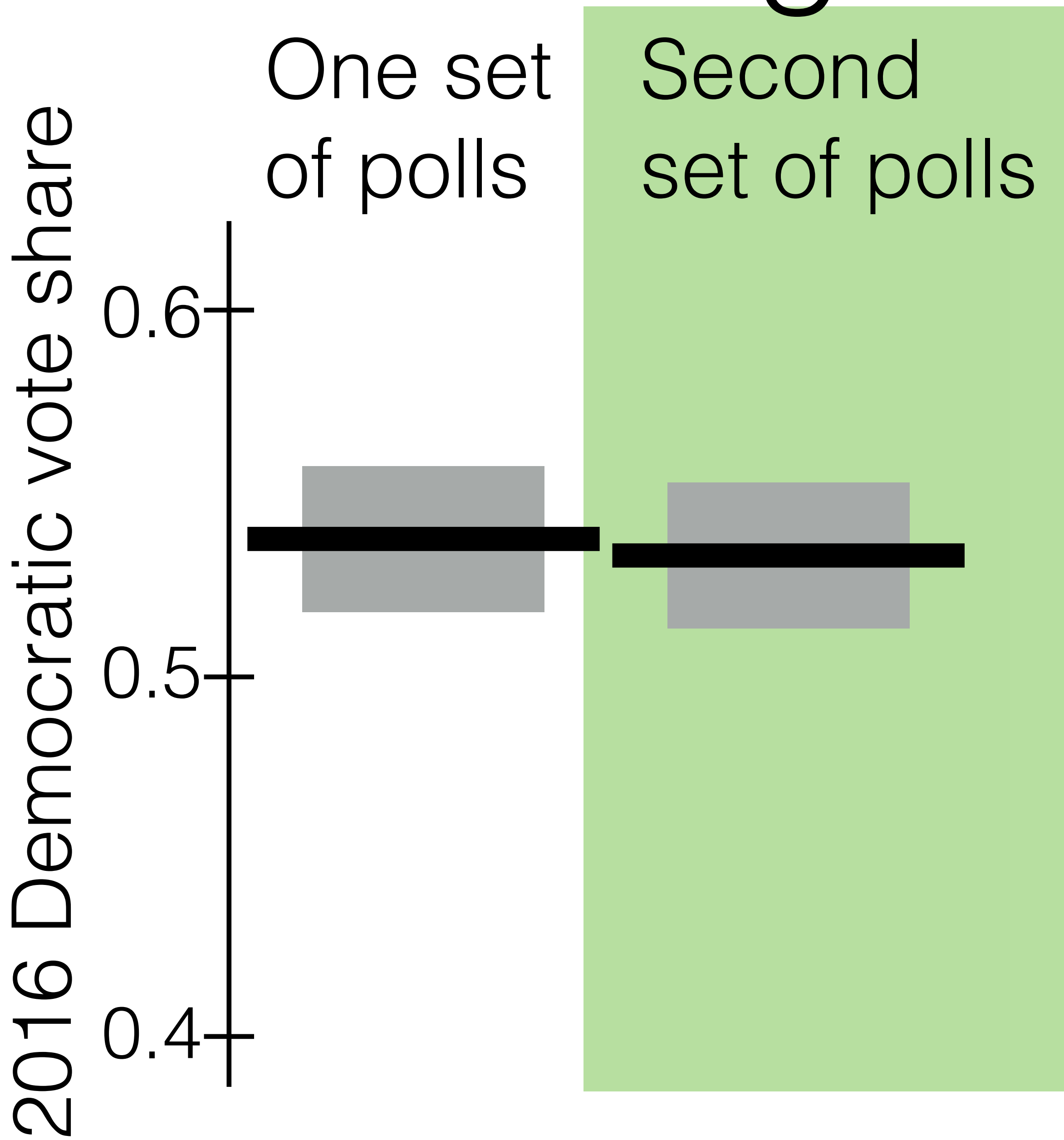

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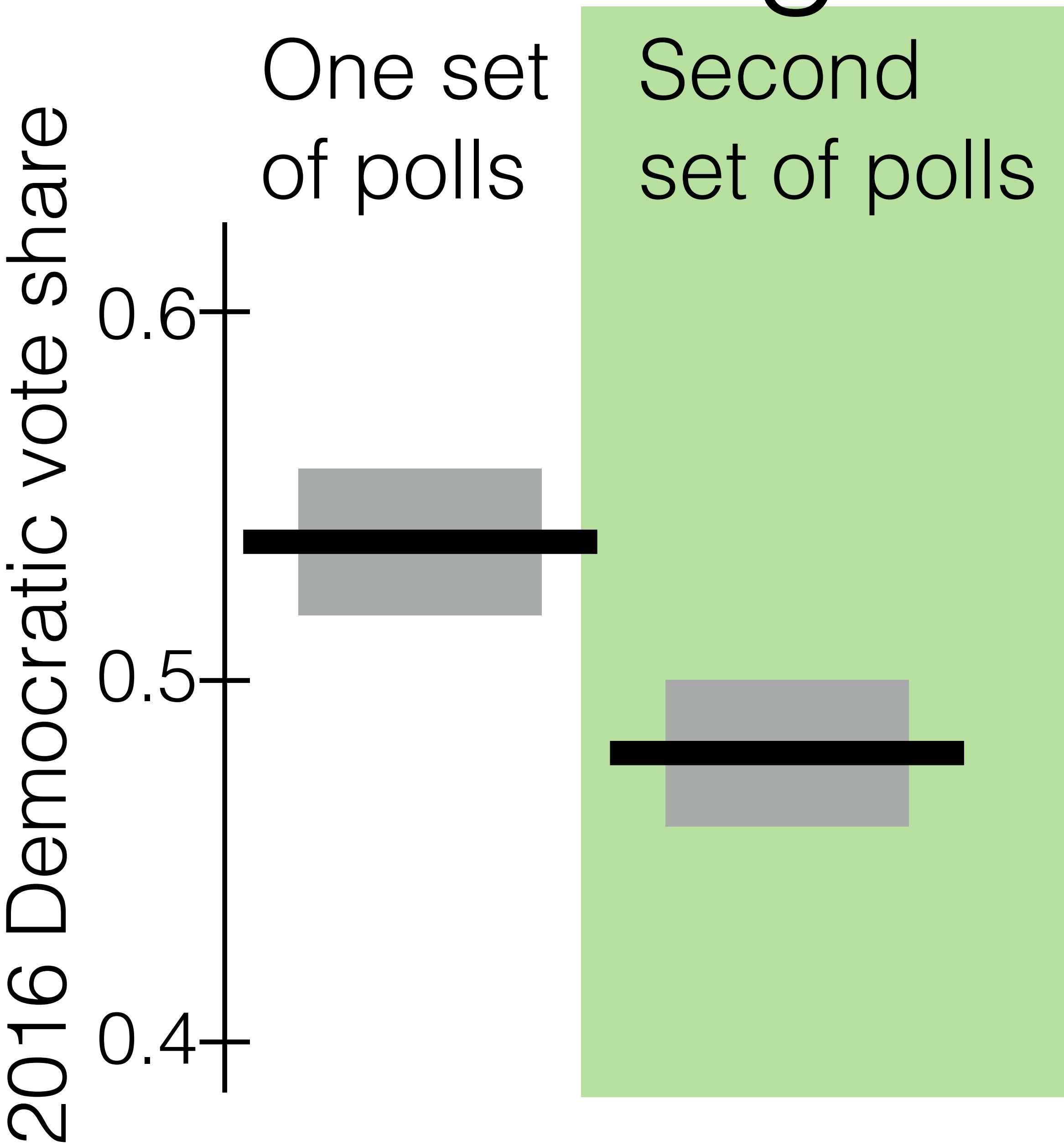

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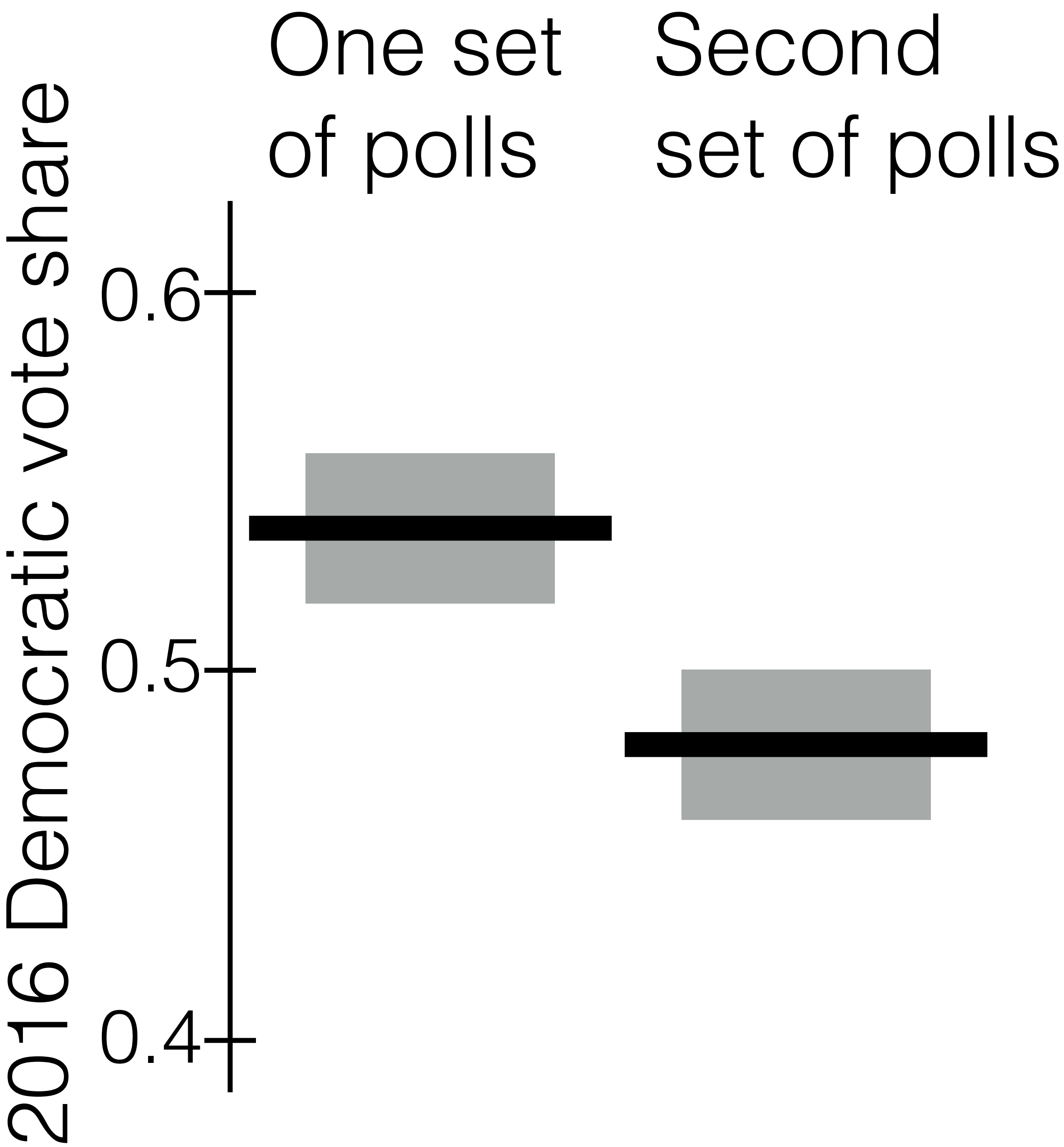

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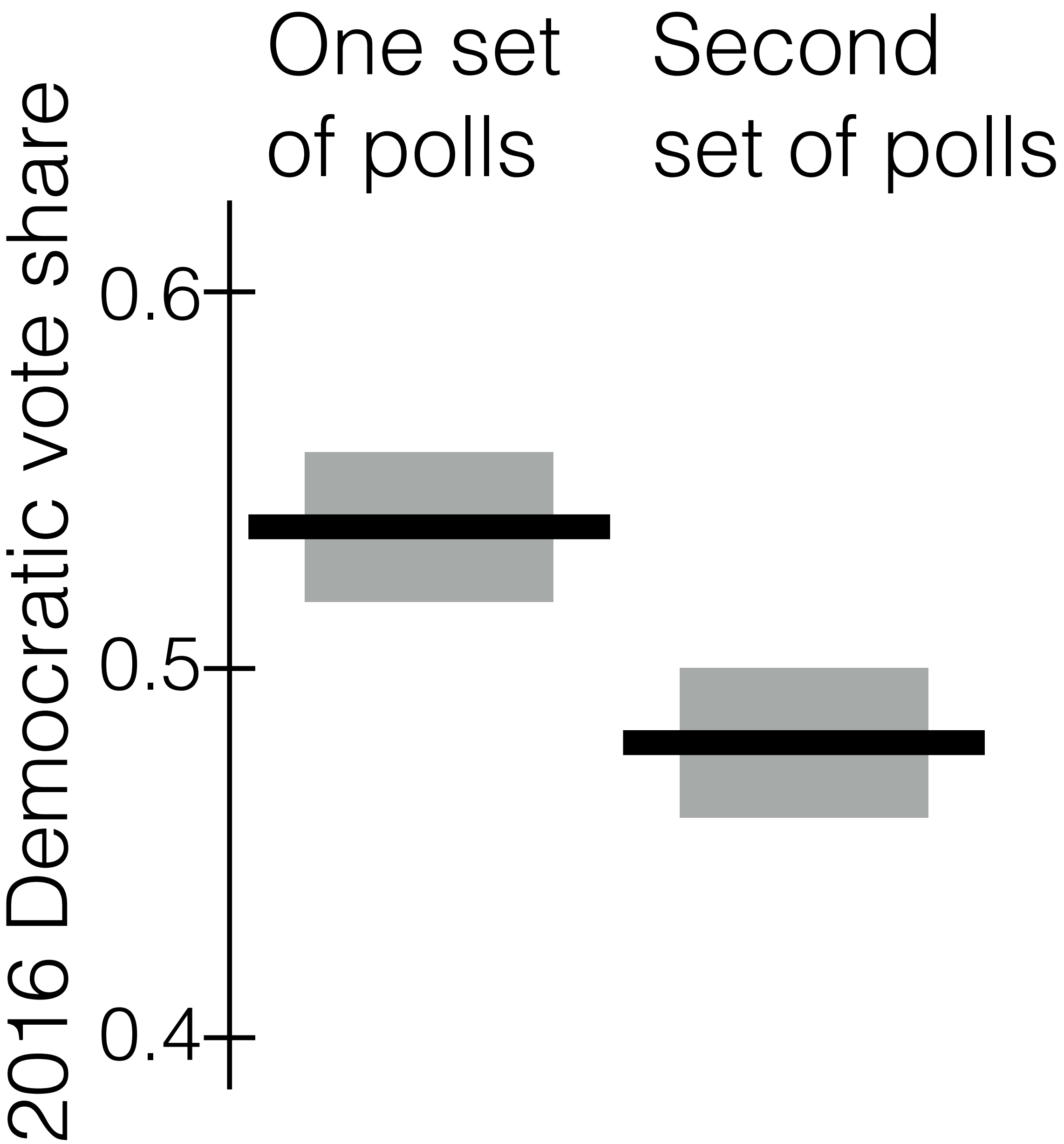

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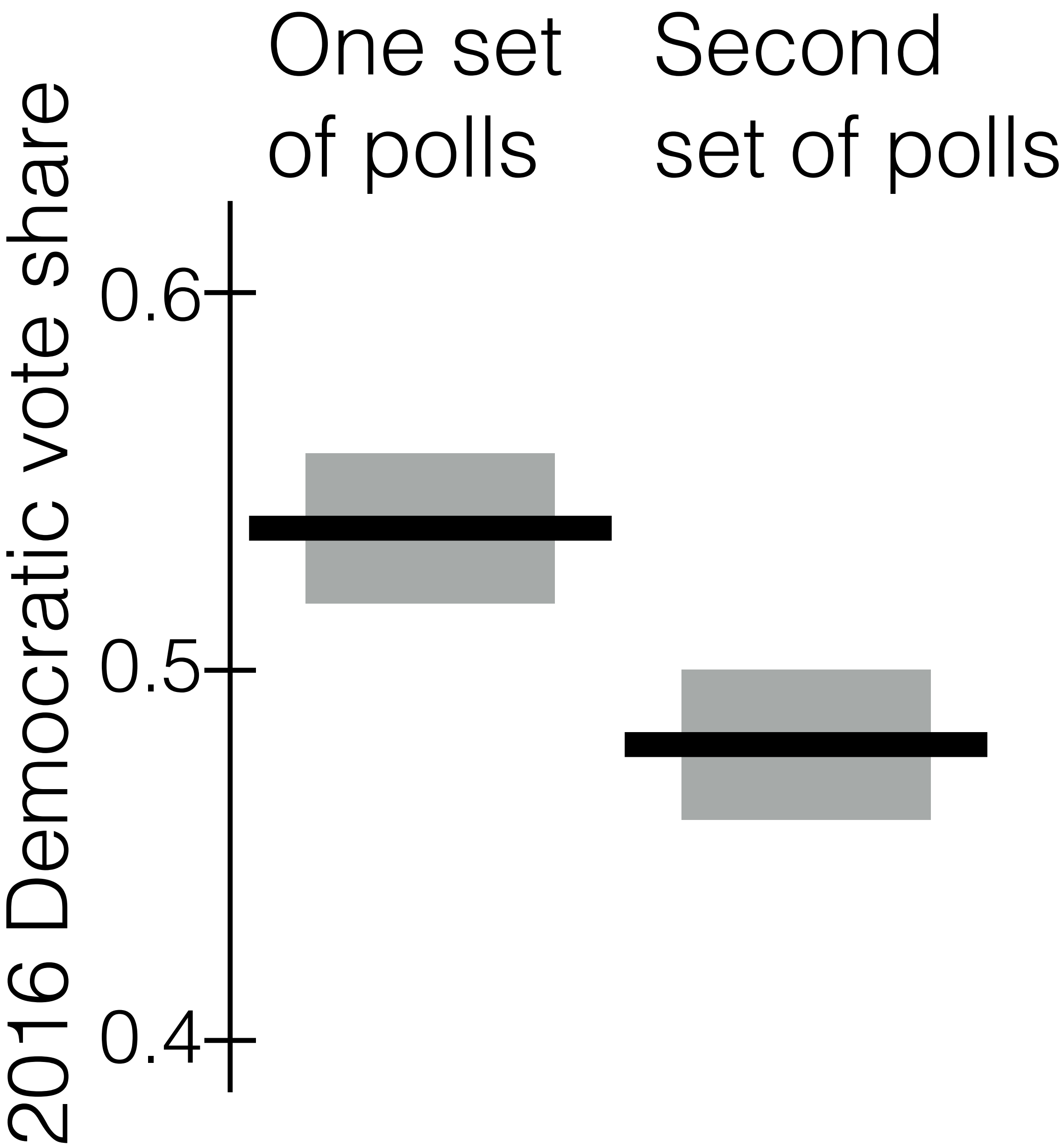

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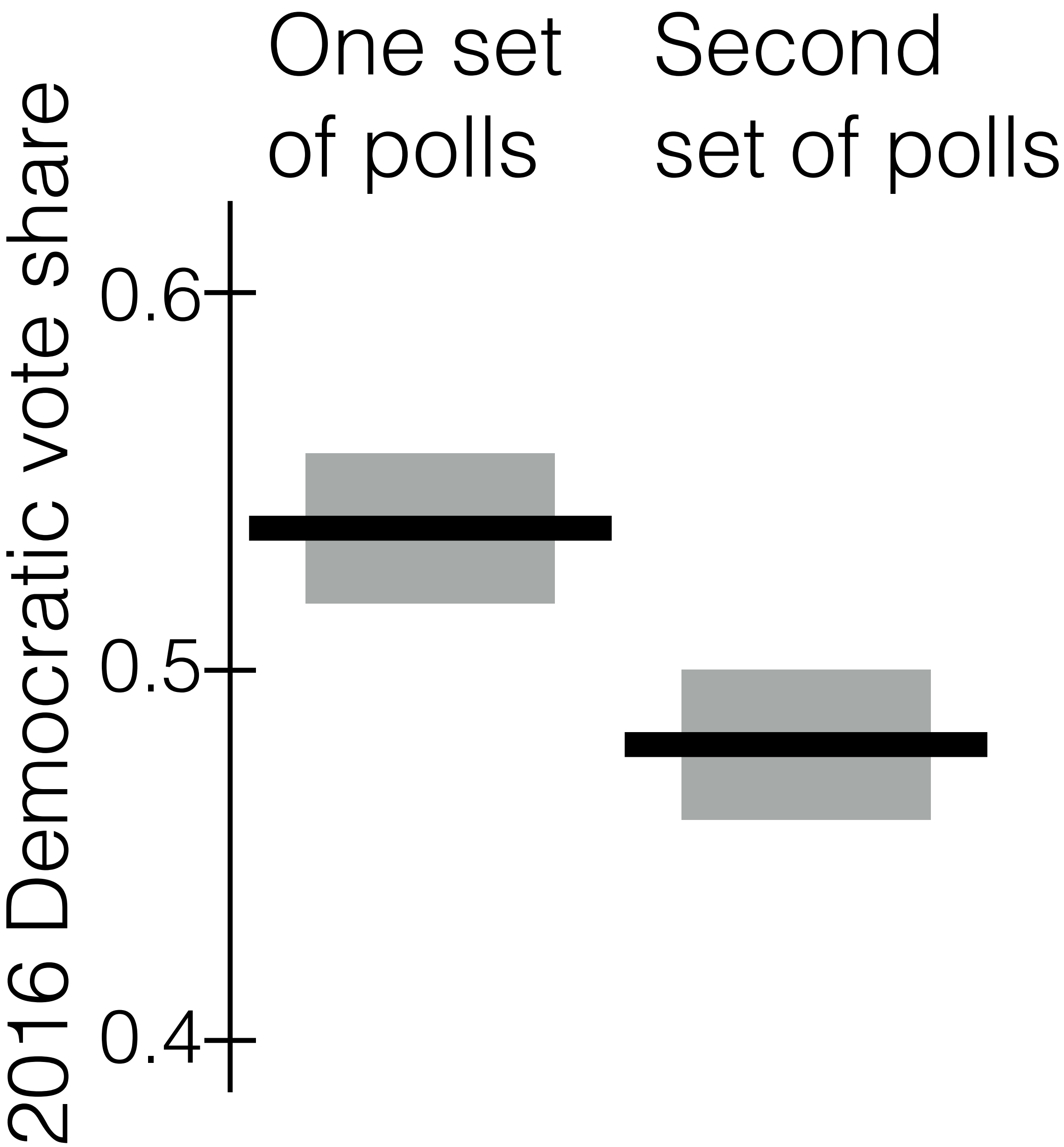

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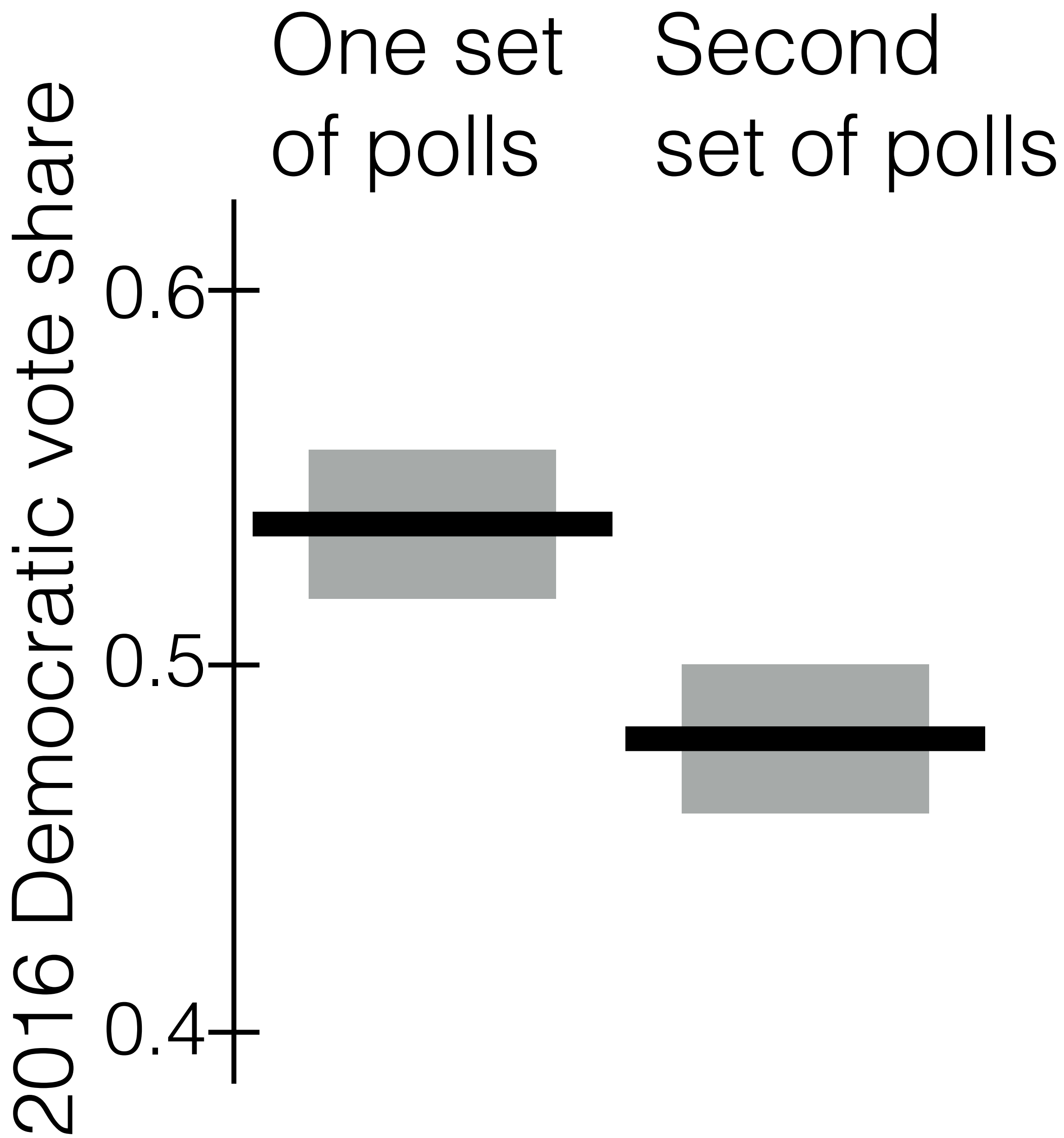

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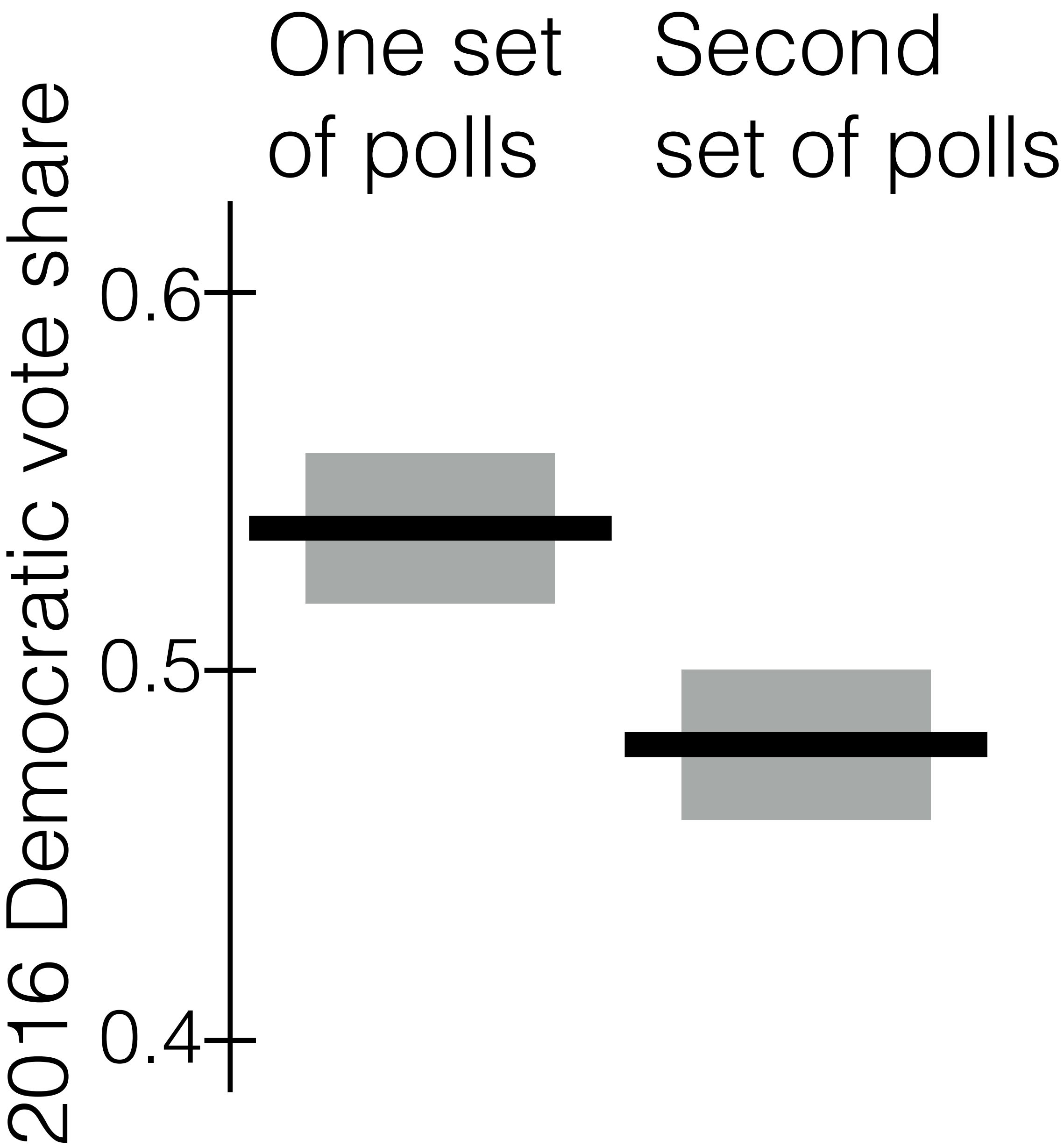

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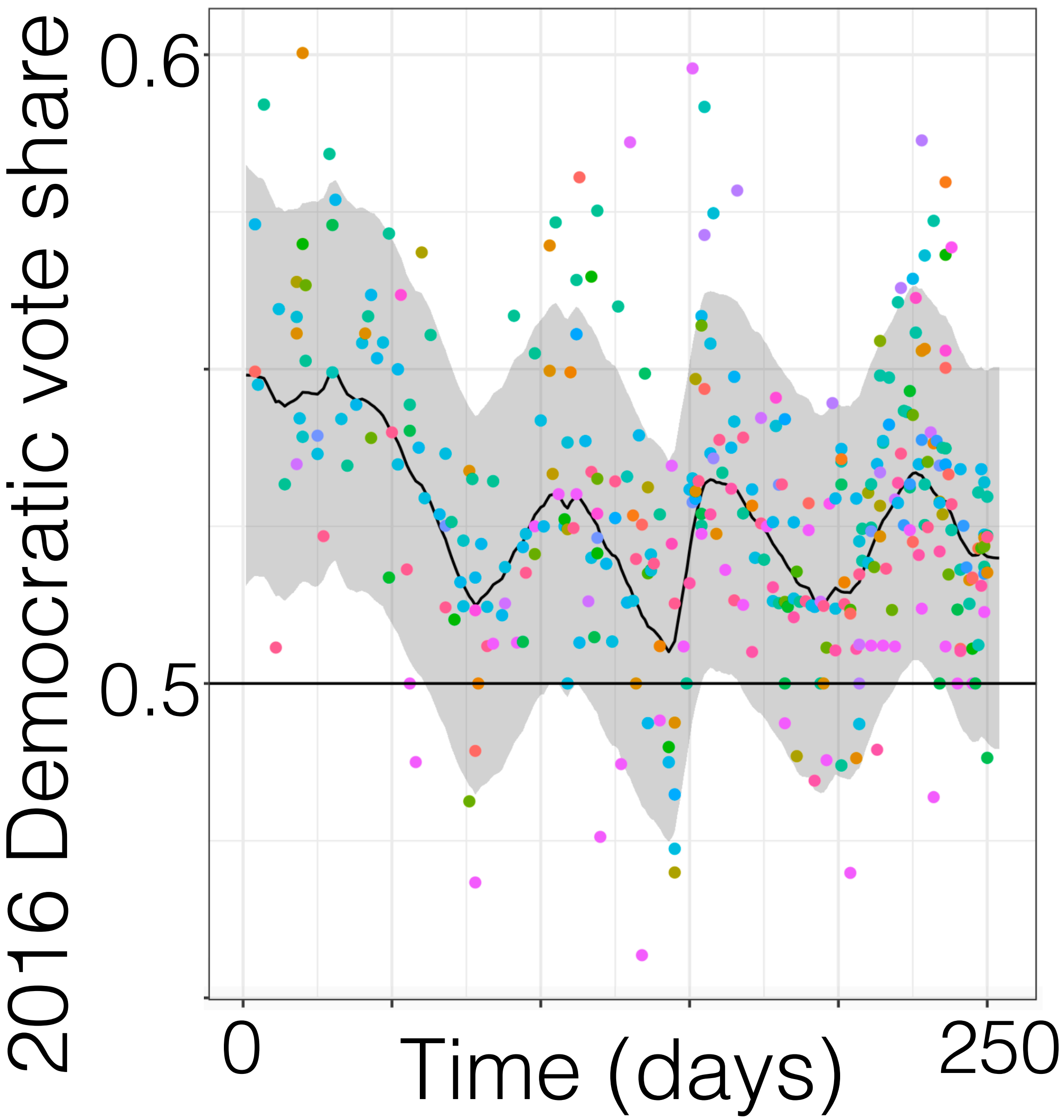

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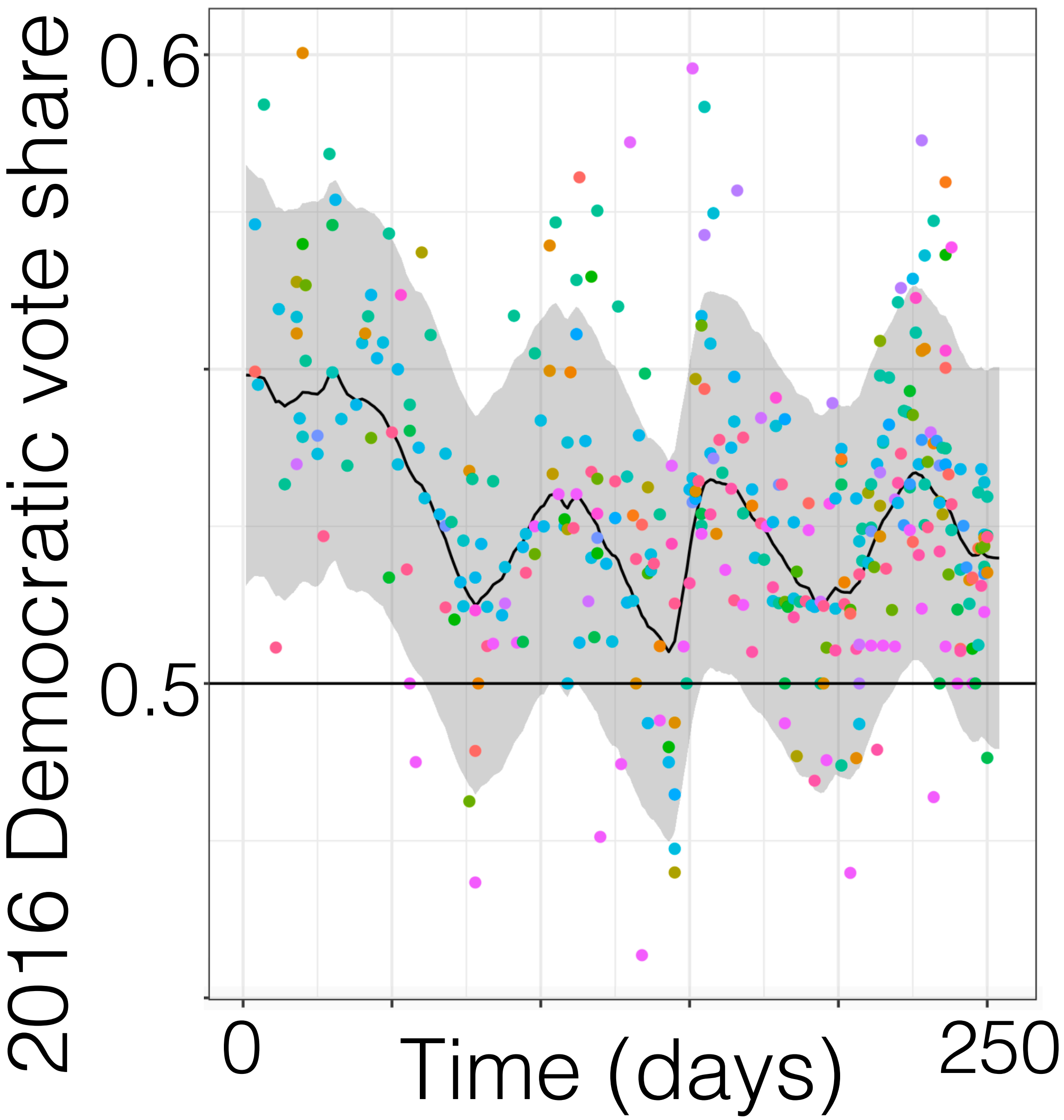

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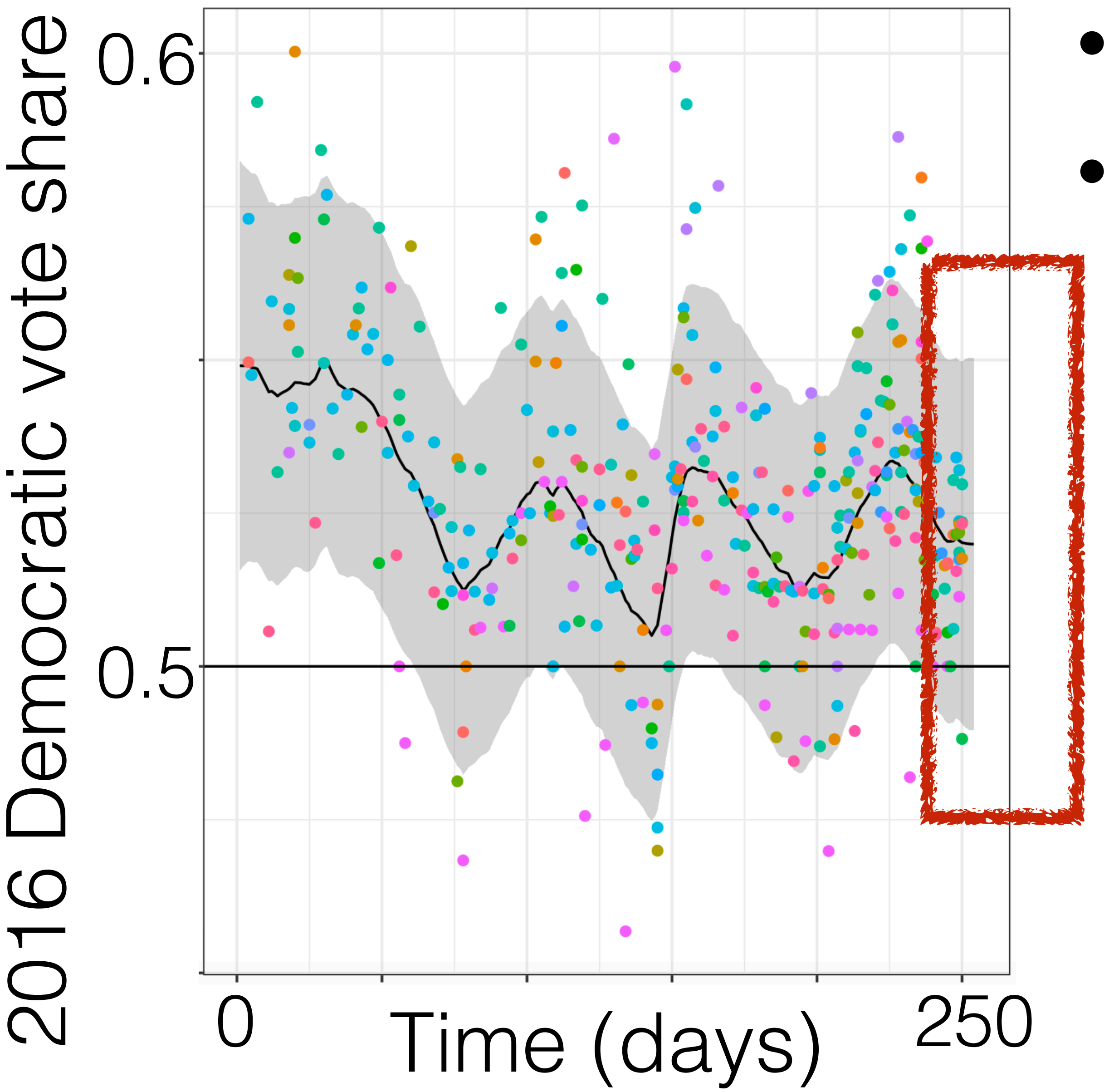

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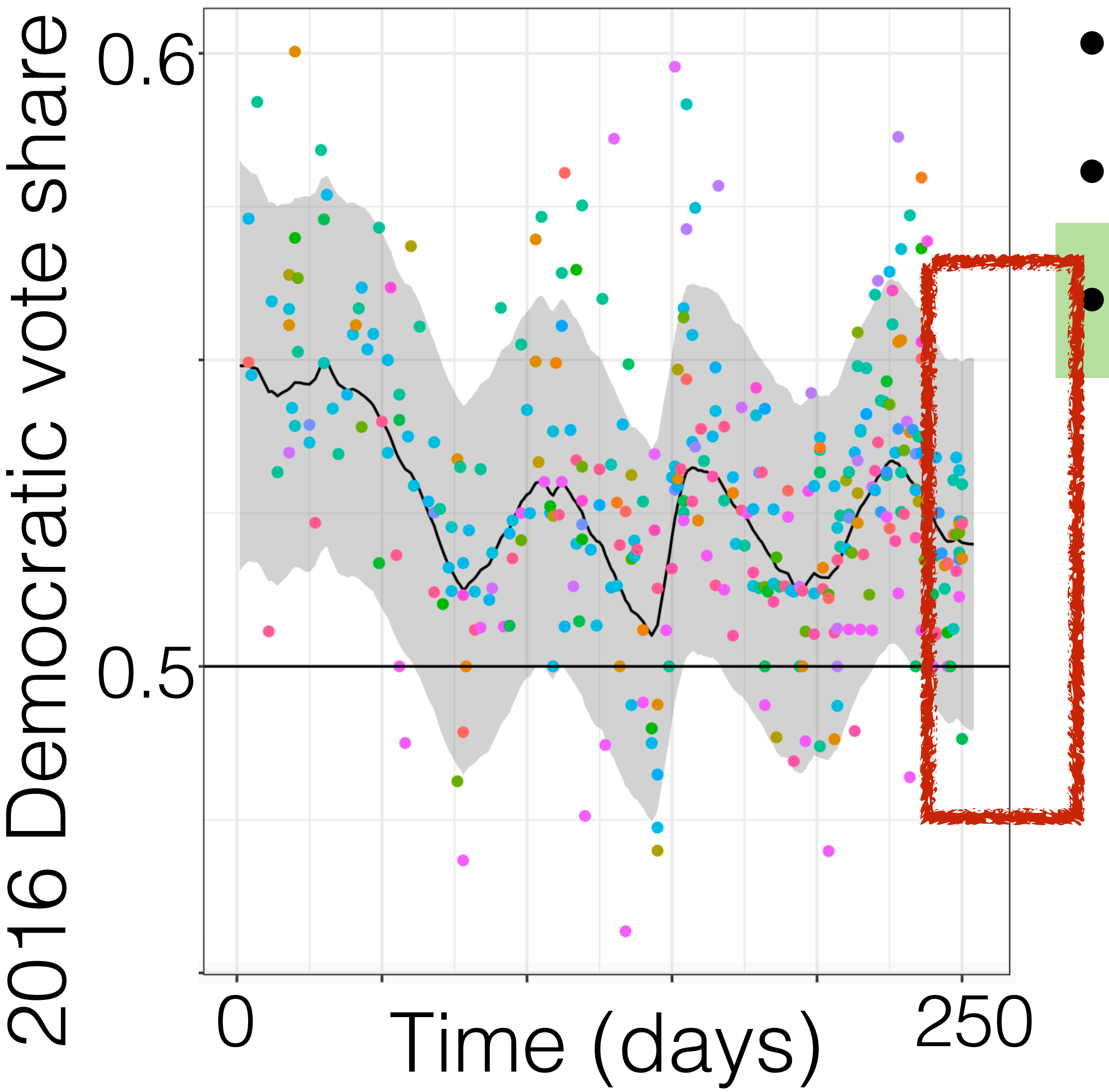

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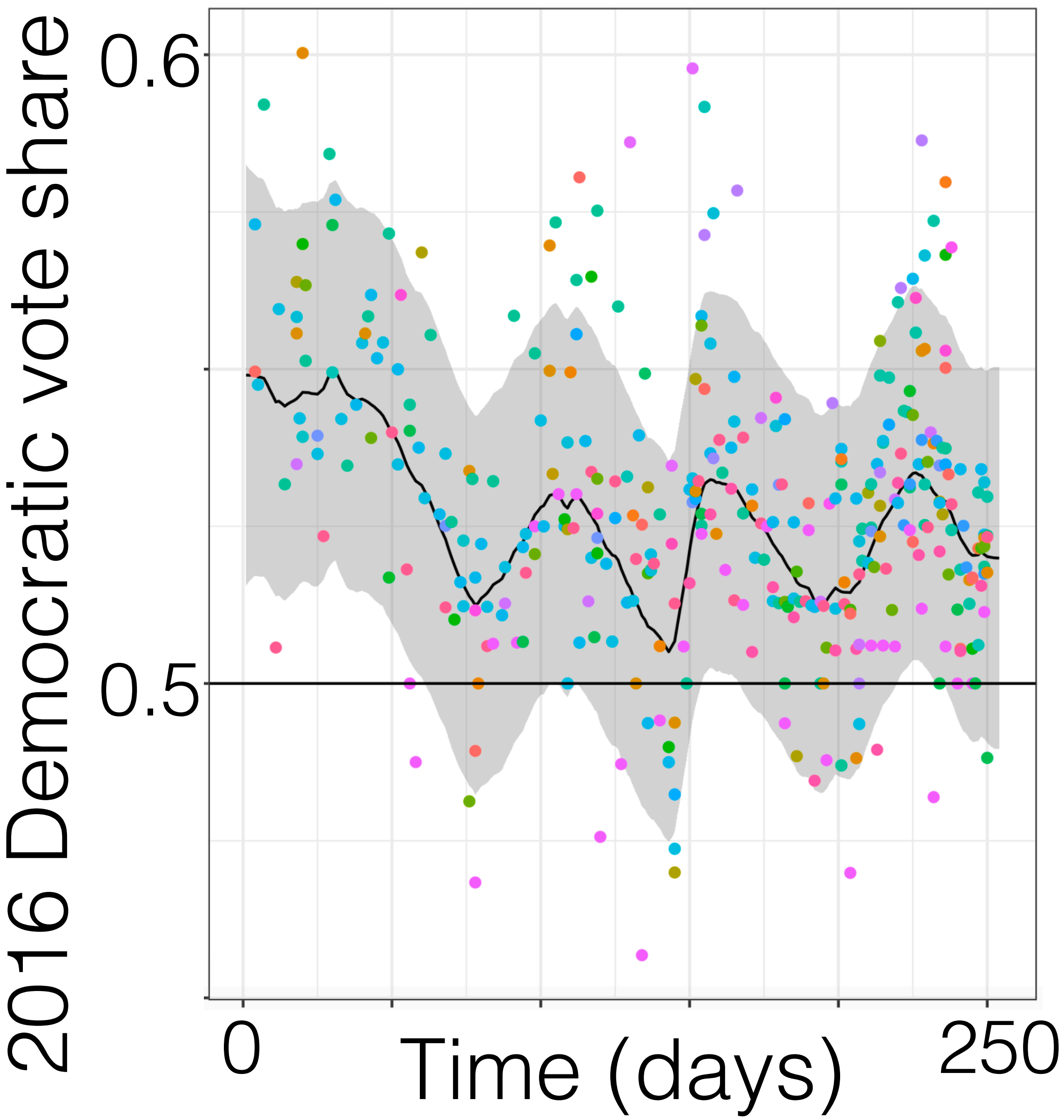

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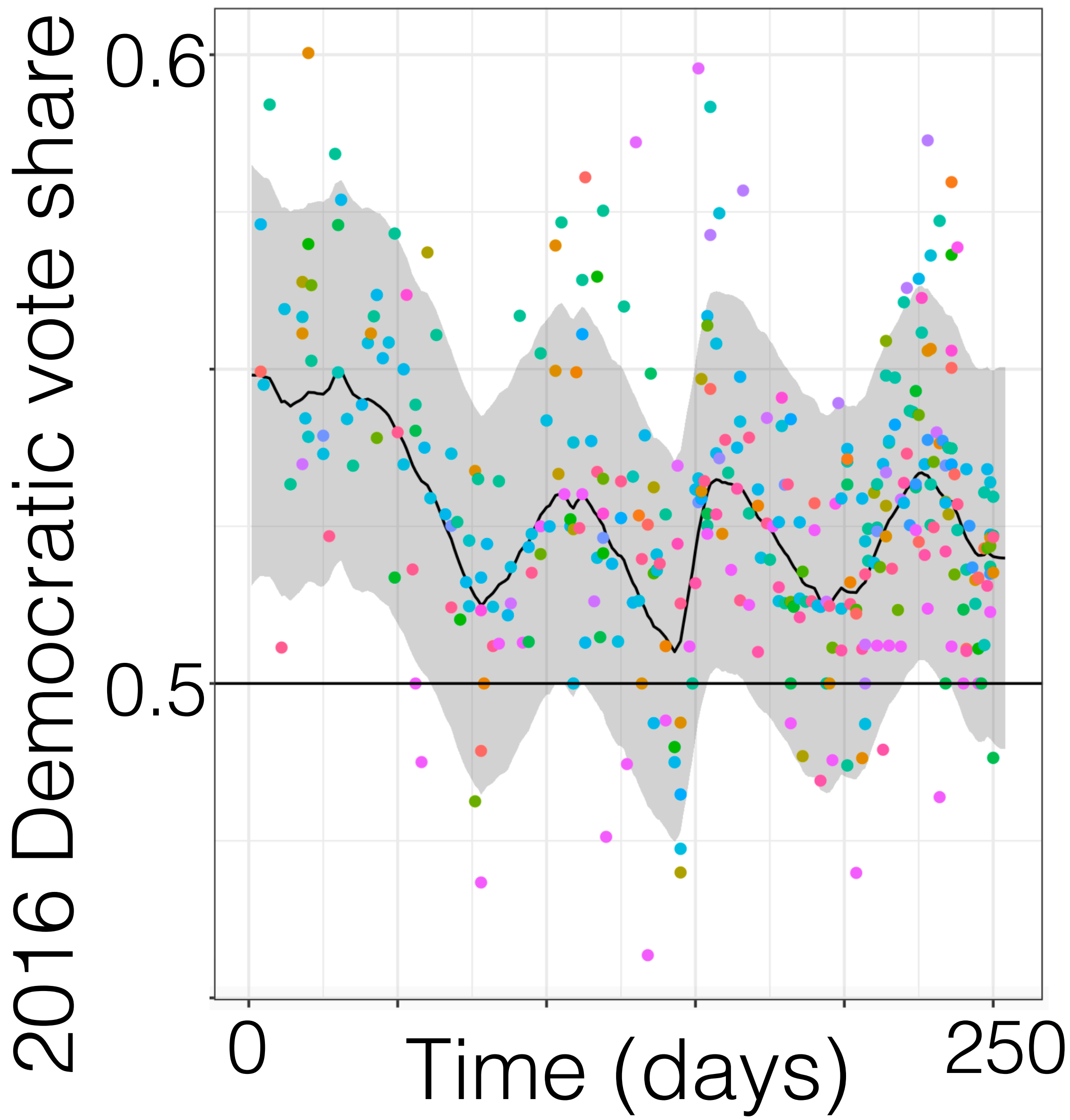

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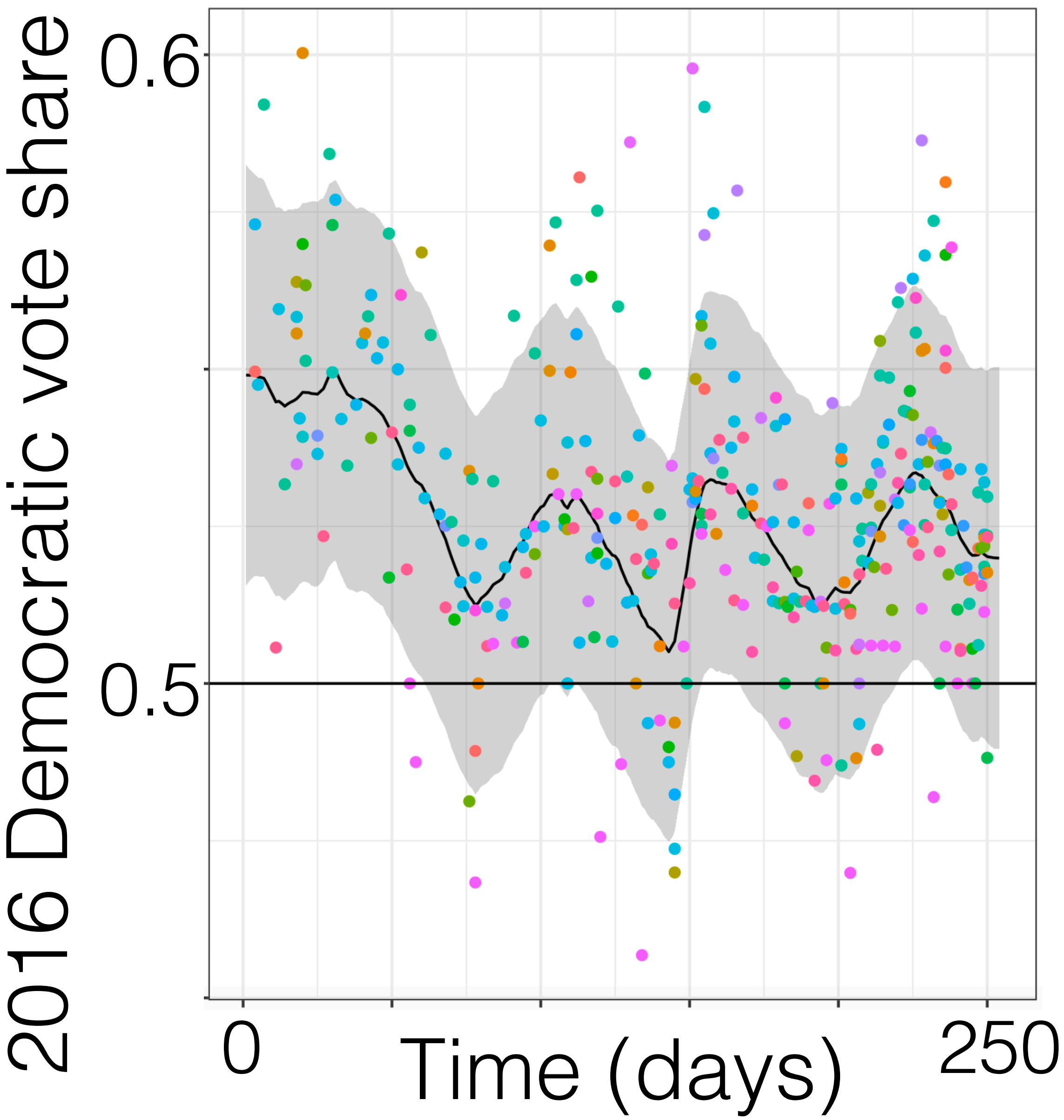

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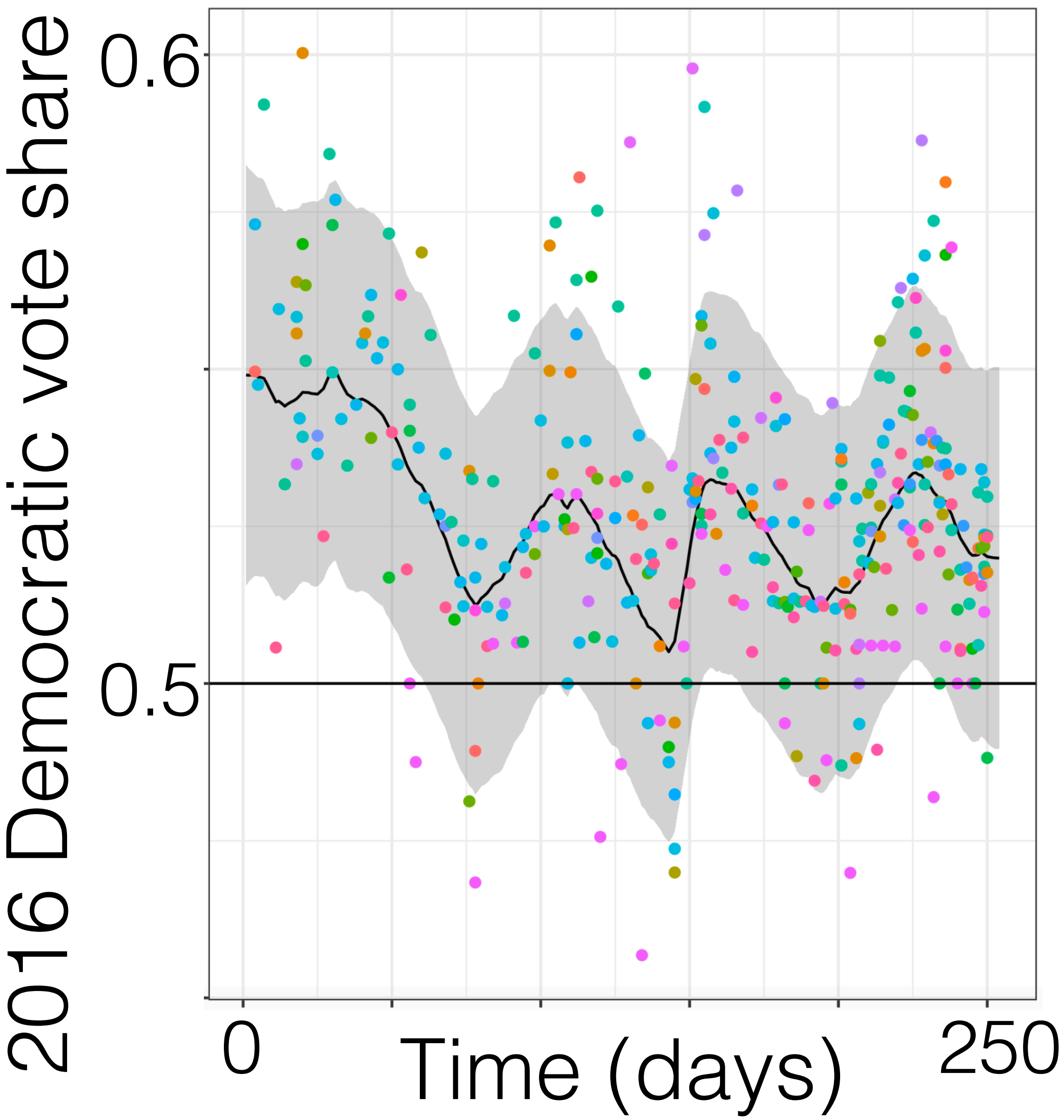

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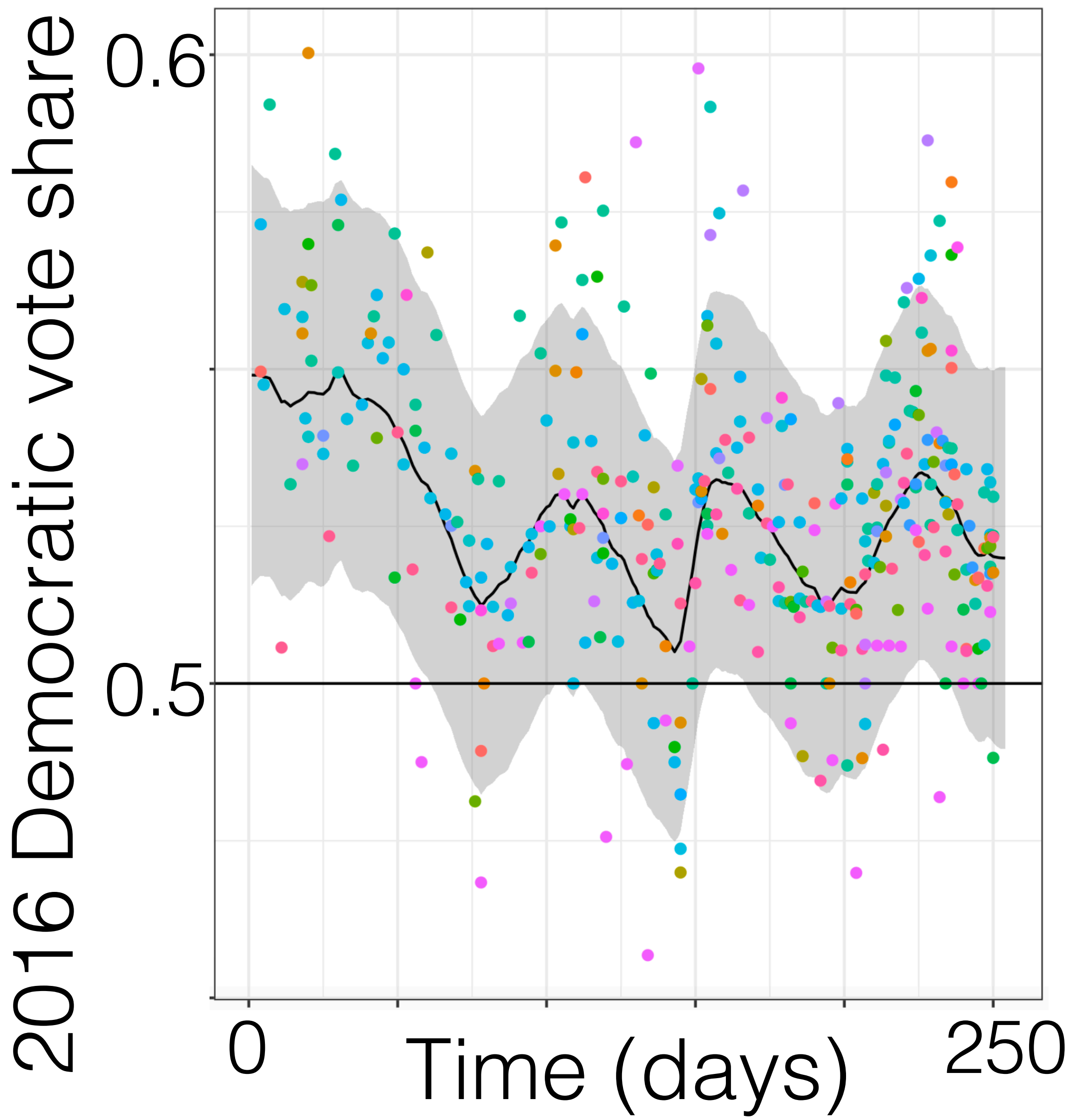

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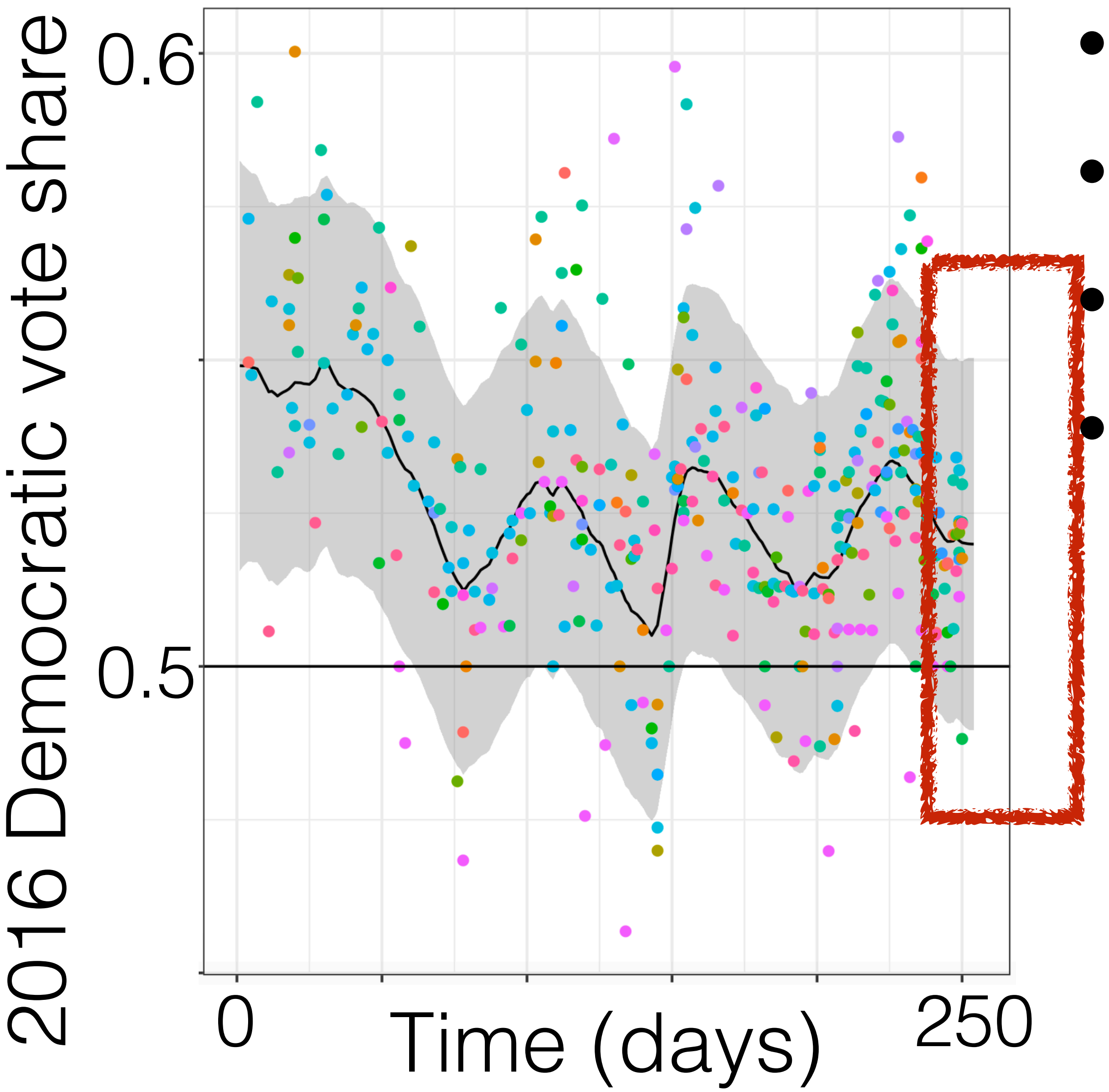

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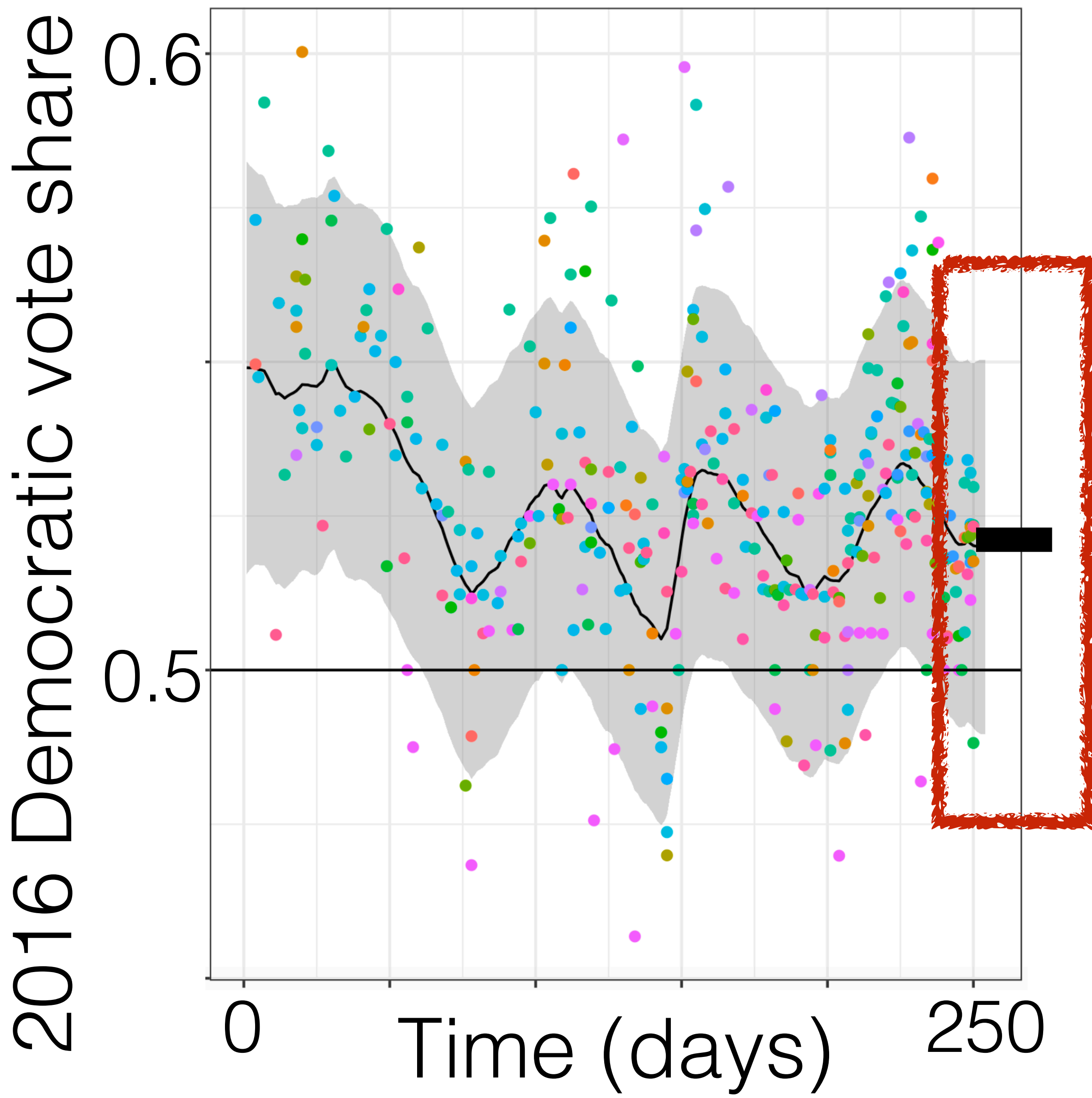

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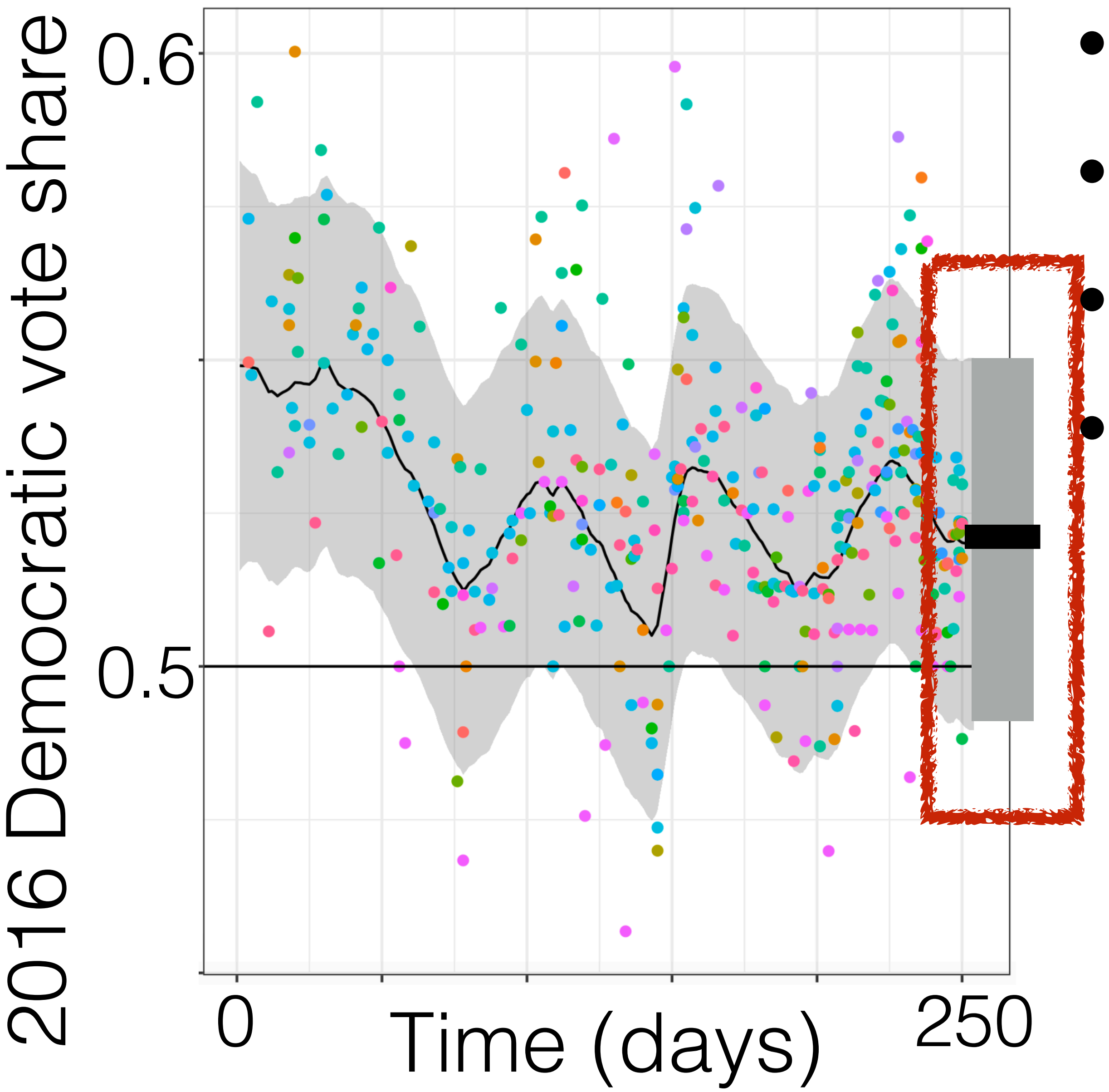

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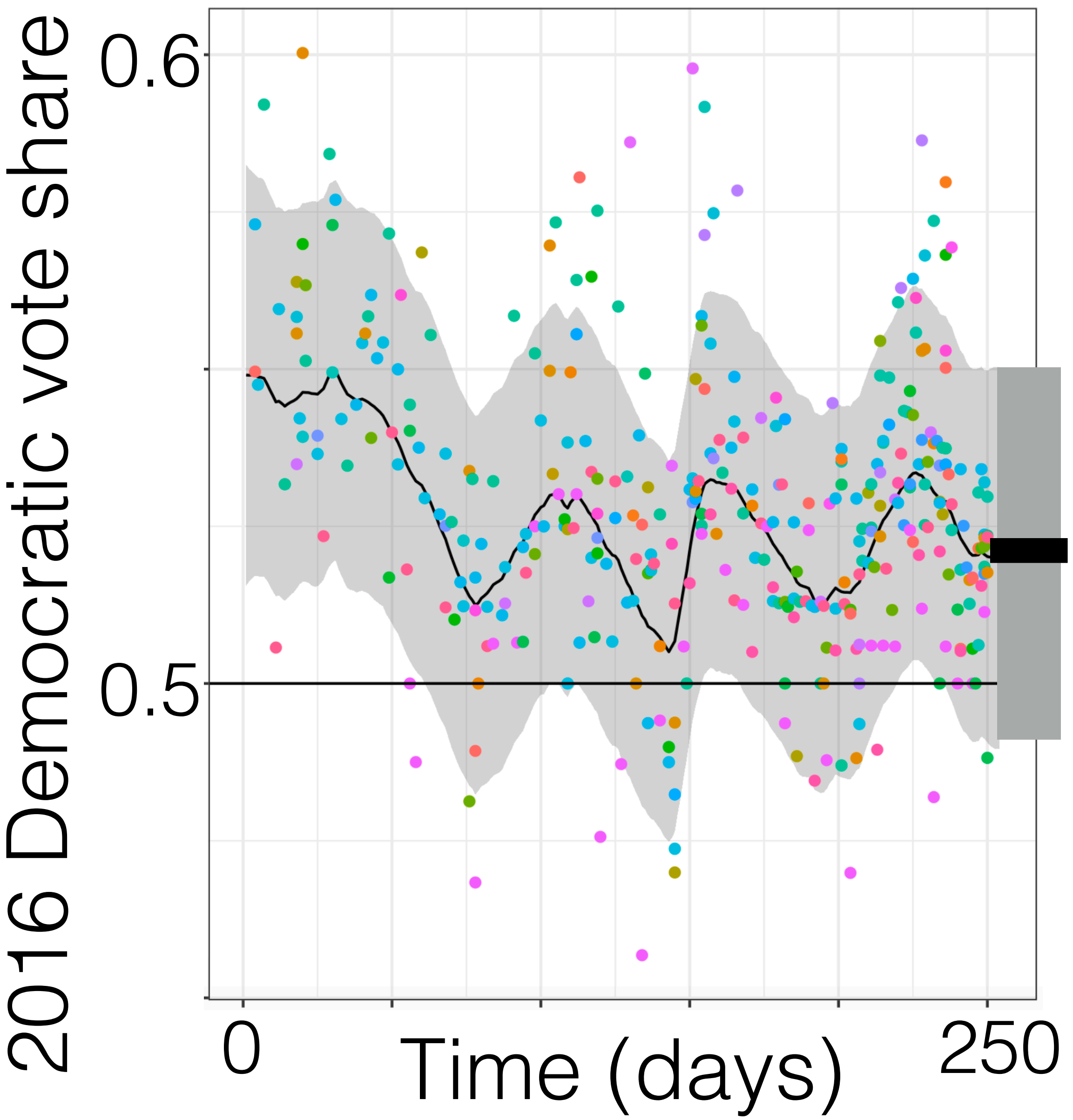

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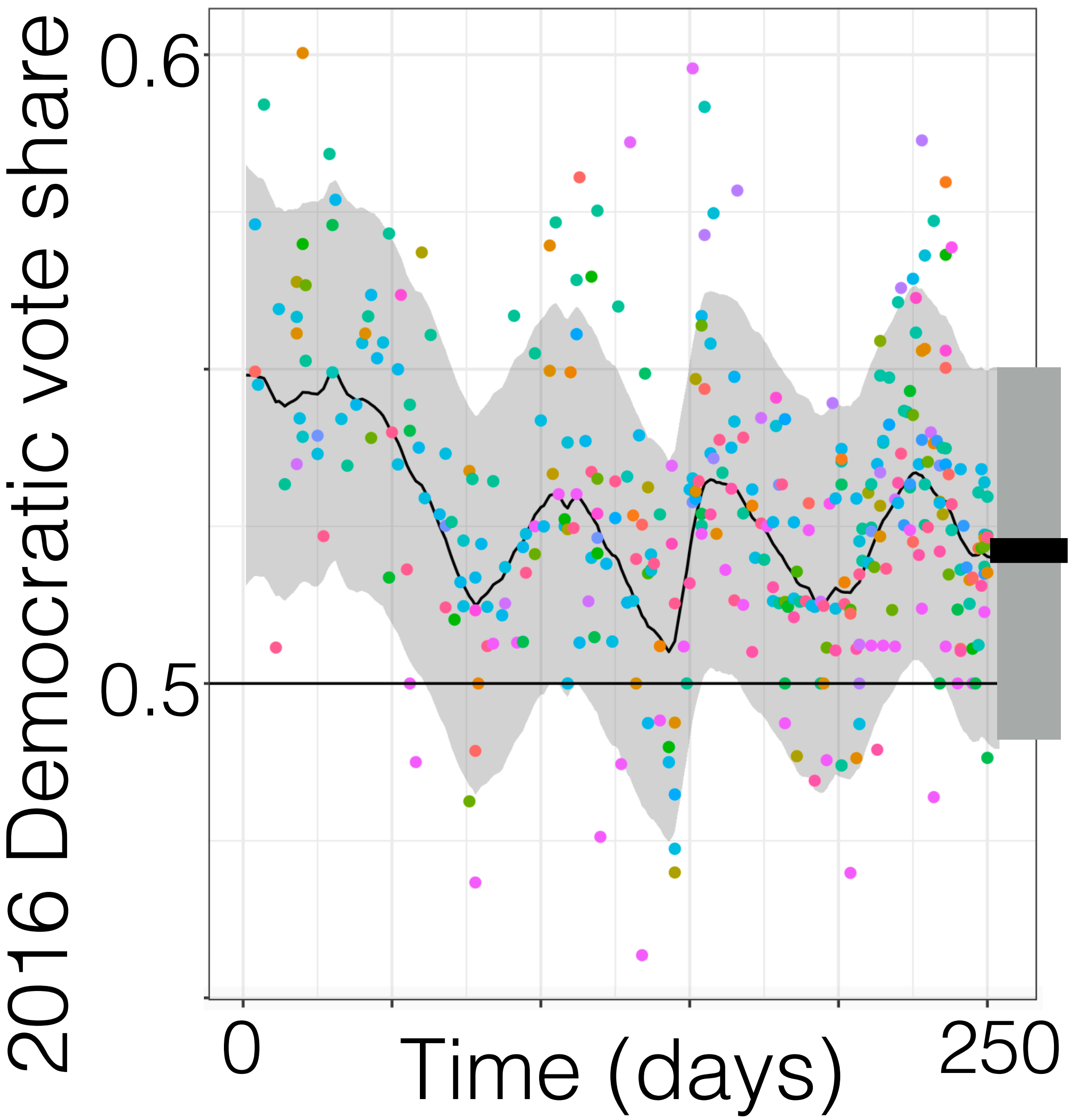

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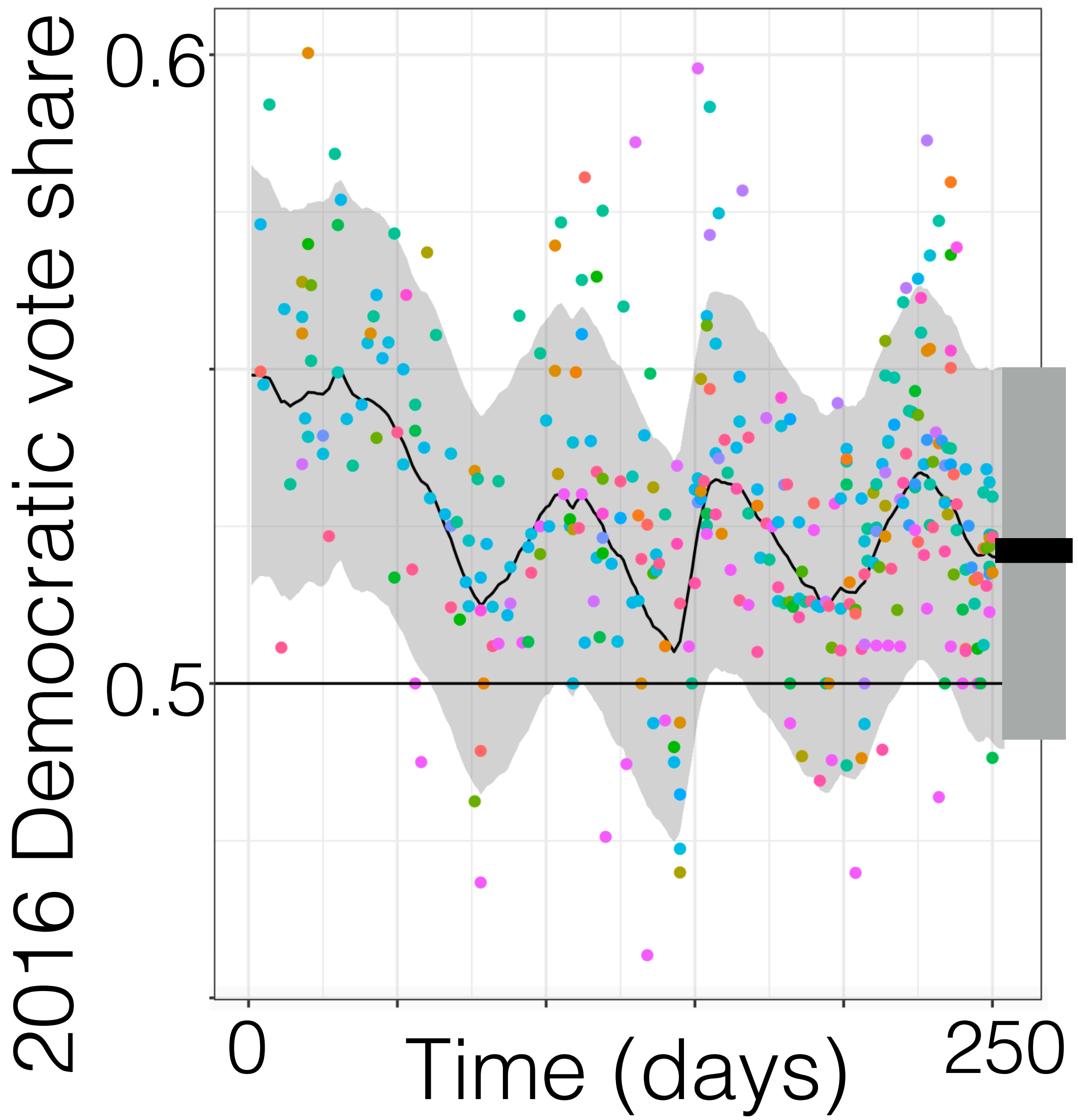

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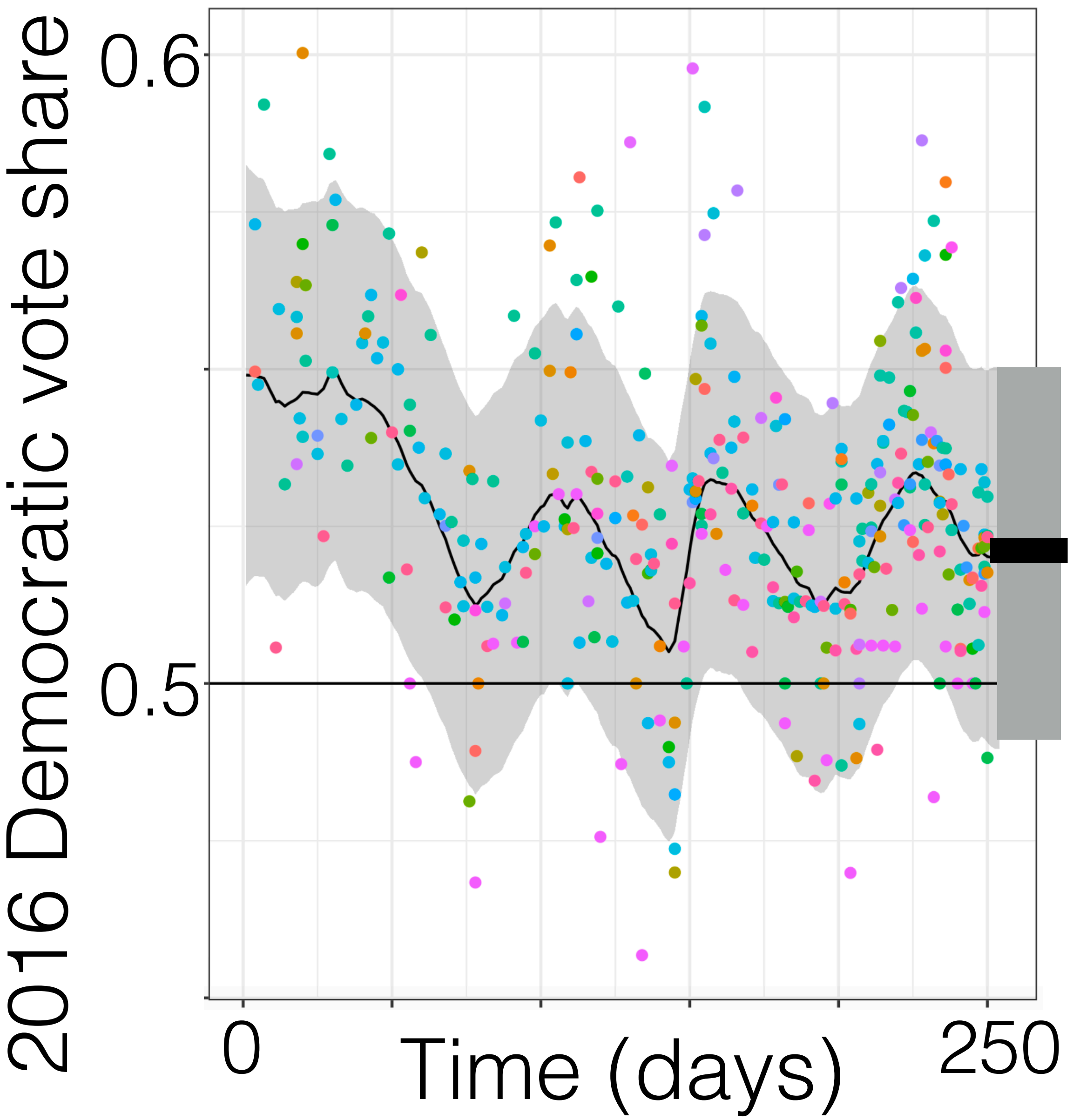

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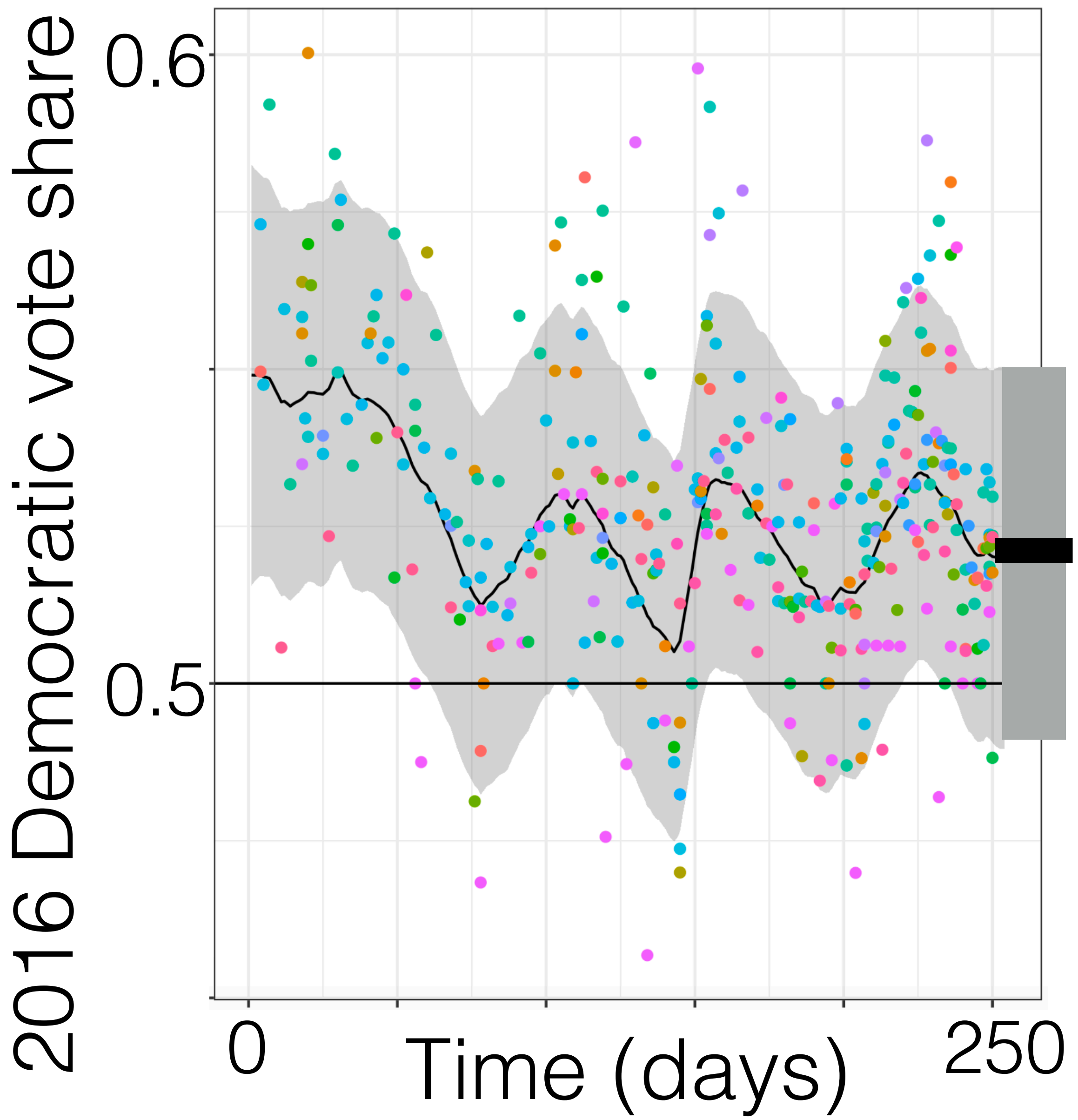

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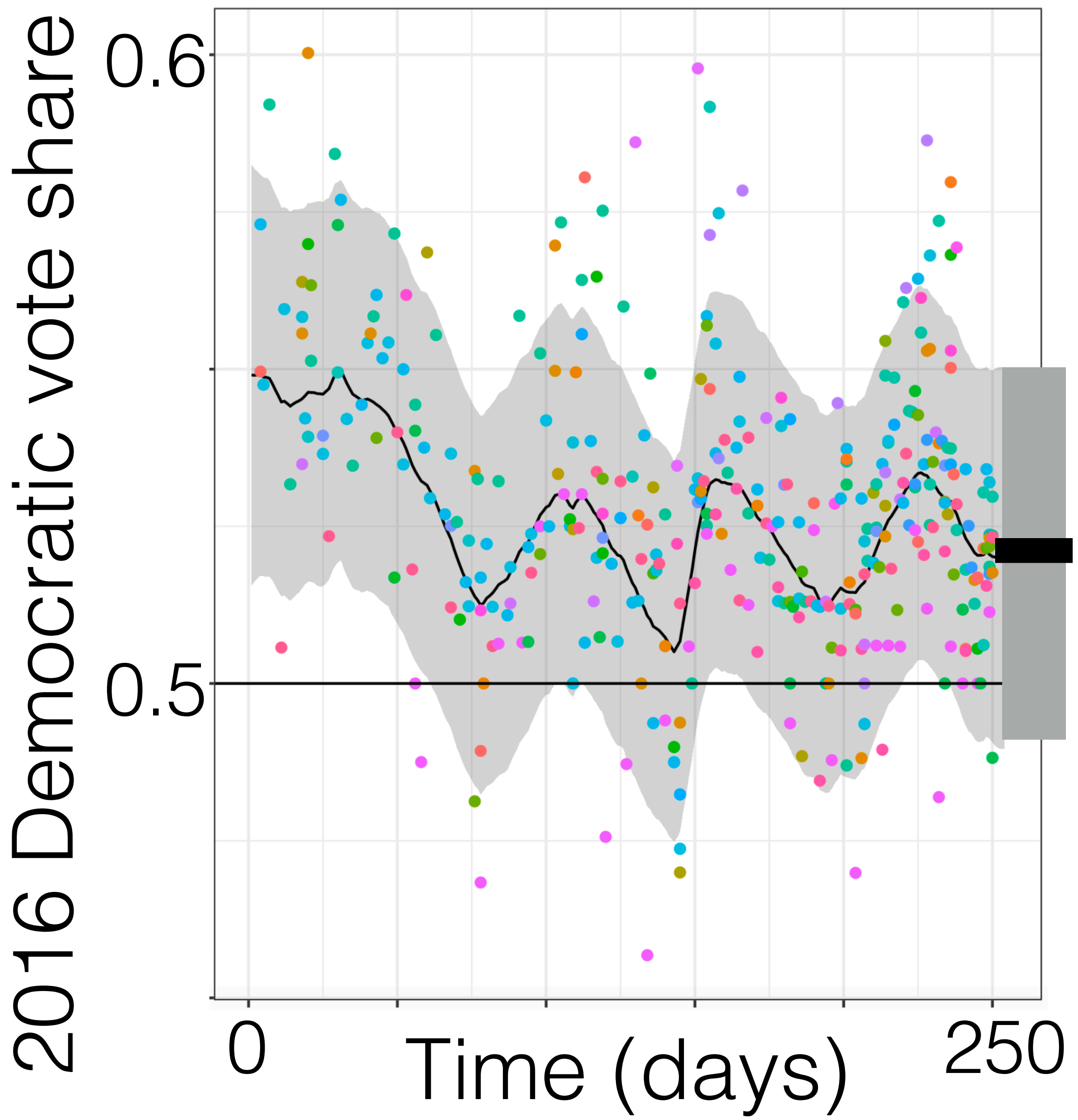

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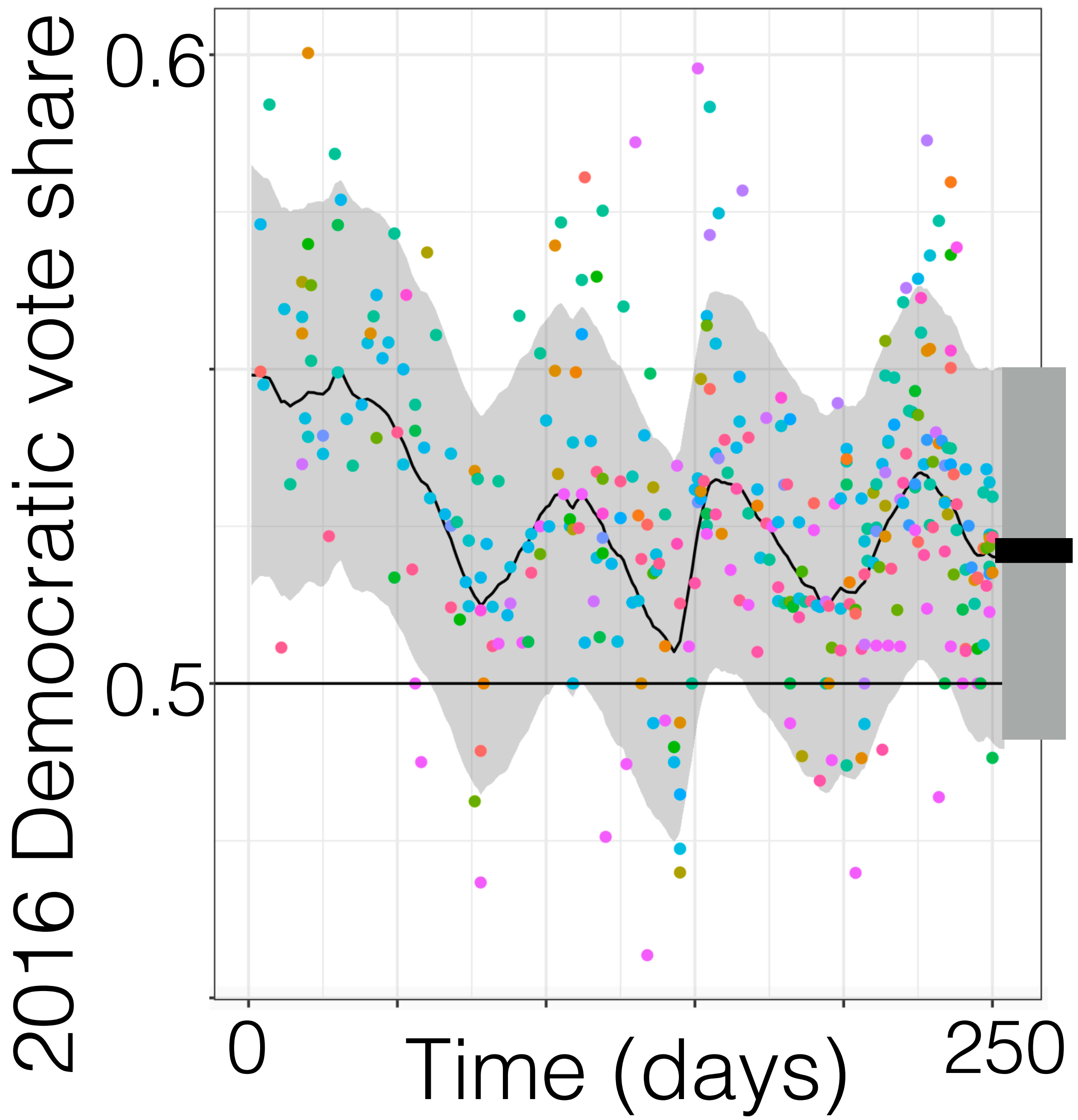

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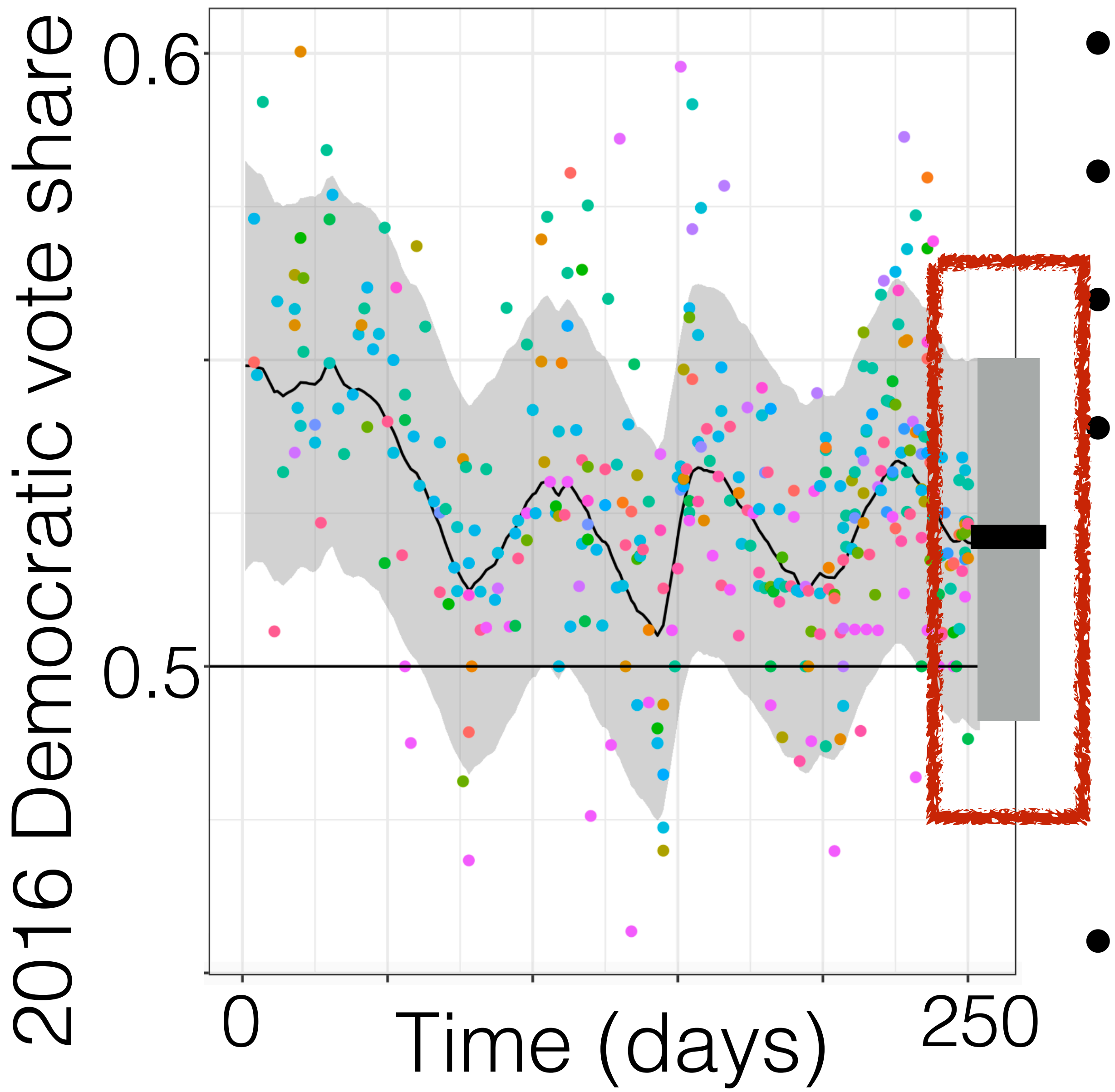

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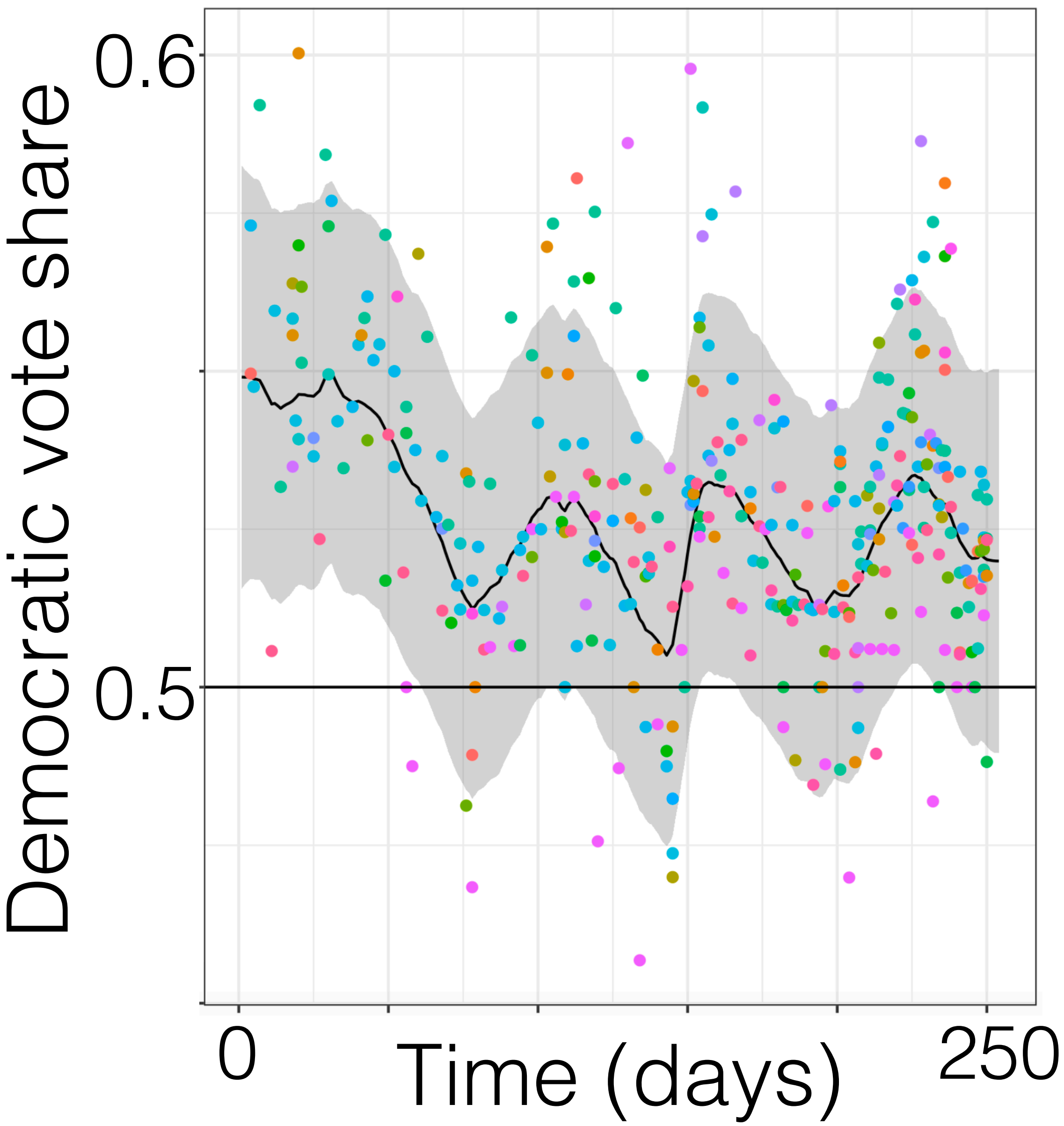
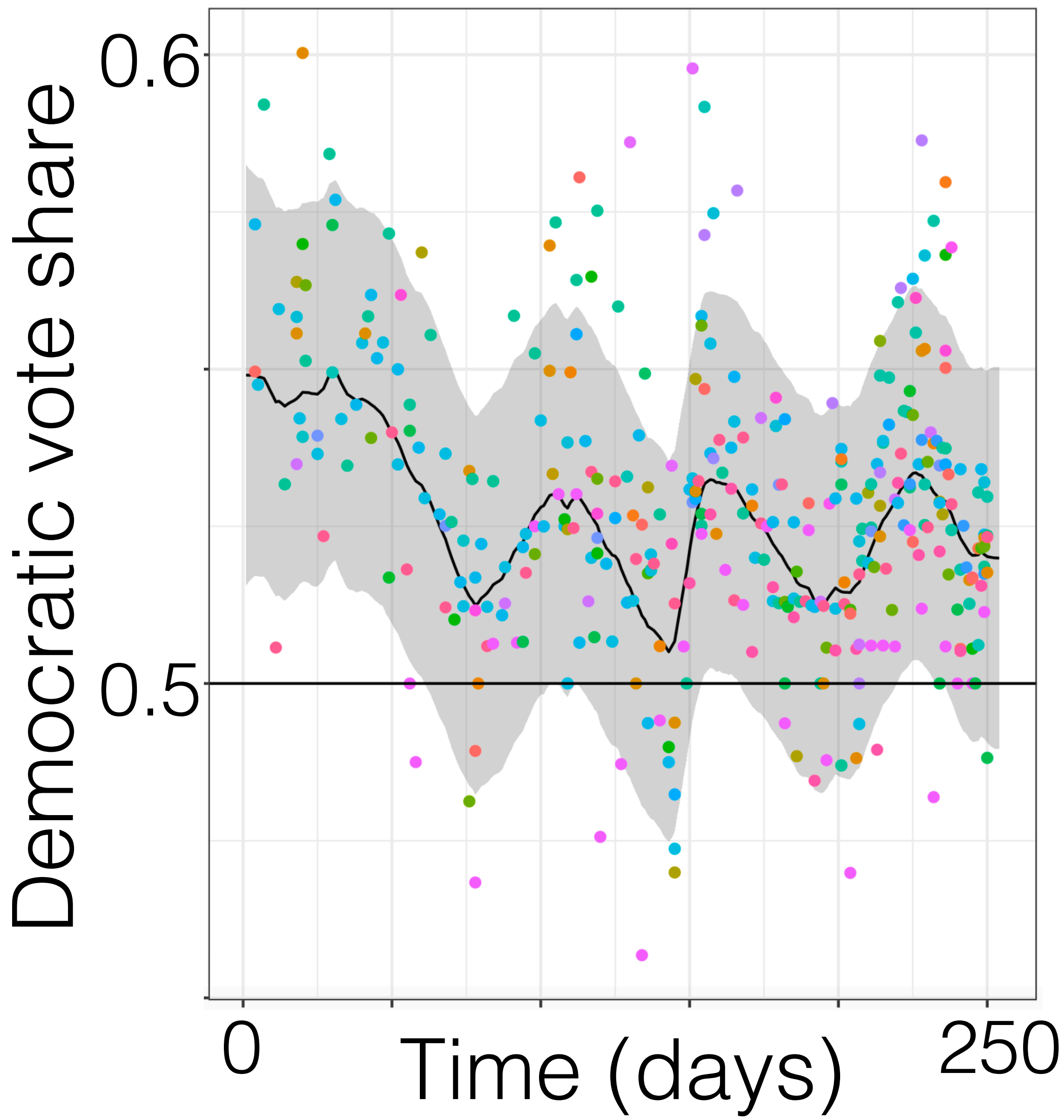


- How to approx $\underset{\mathbb{F}(X)}{\mathrm{Var}} \; \underset{p(\theta|X)}{\mathbb{E}} [g(\theta)]$?

Idea: how about the bootstrap?



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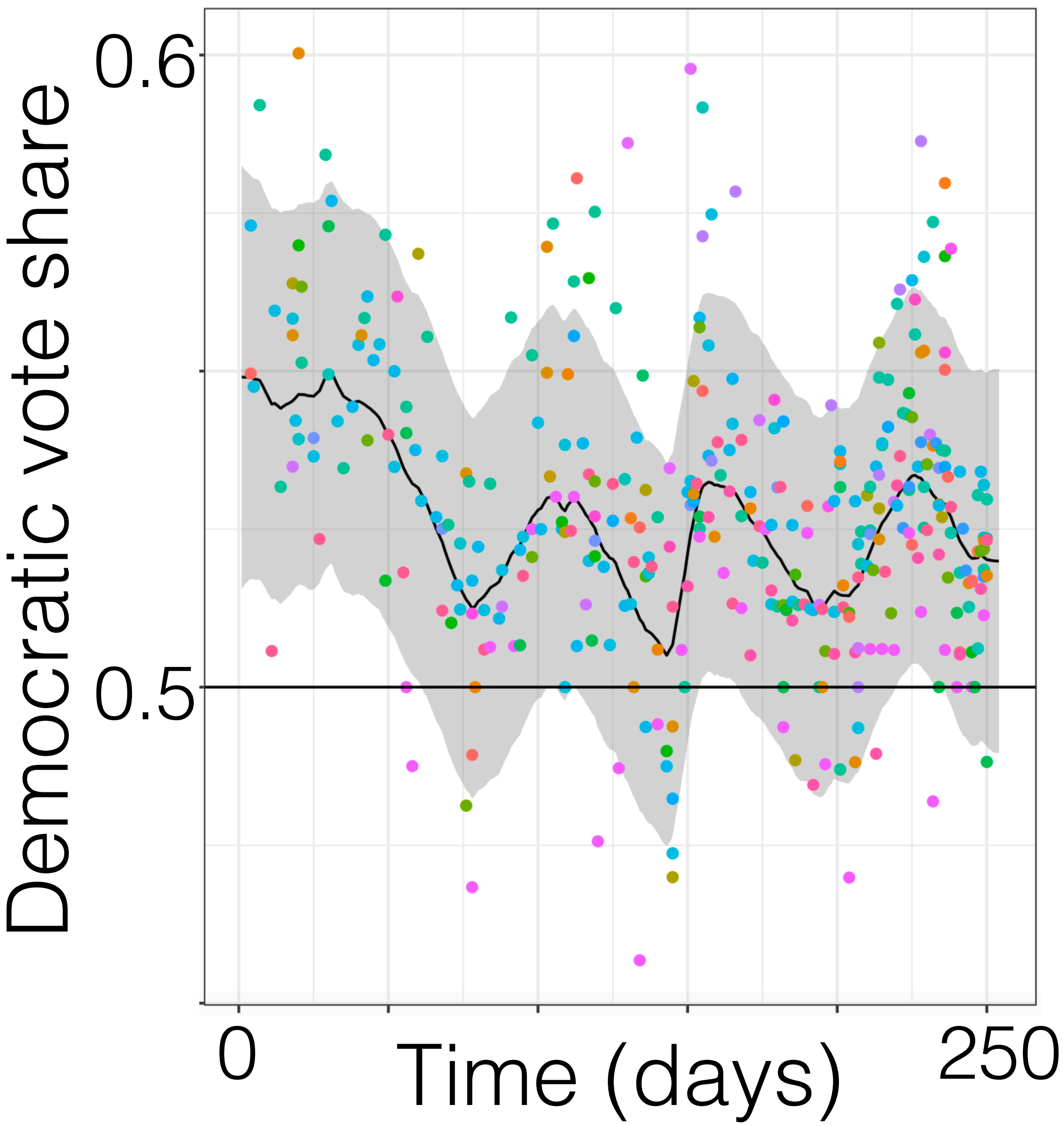

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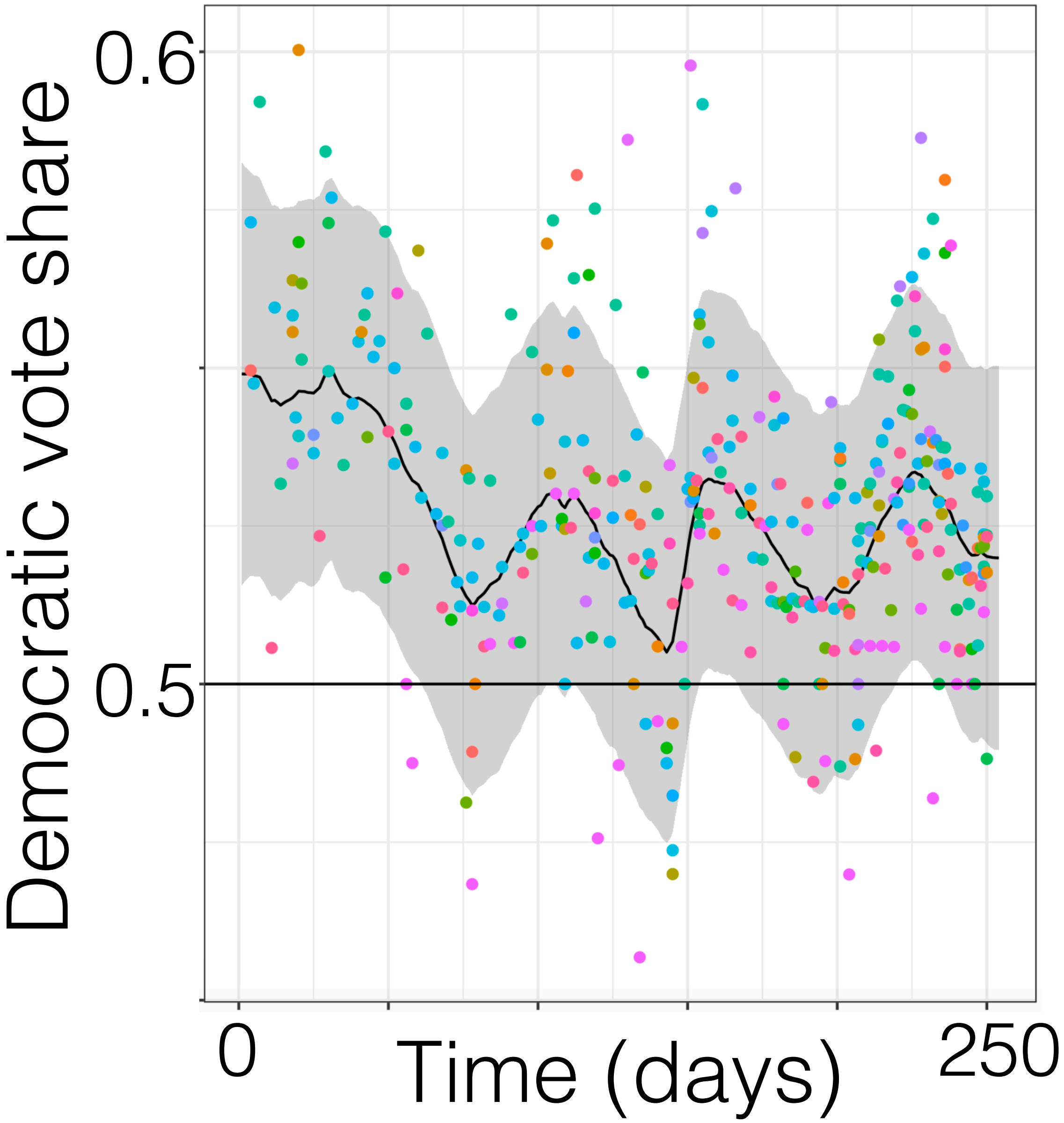

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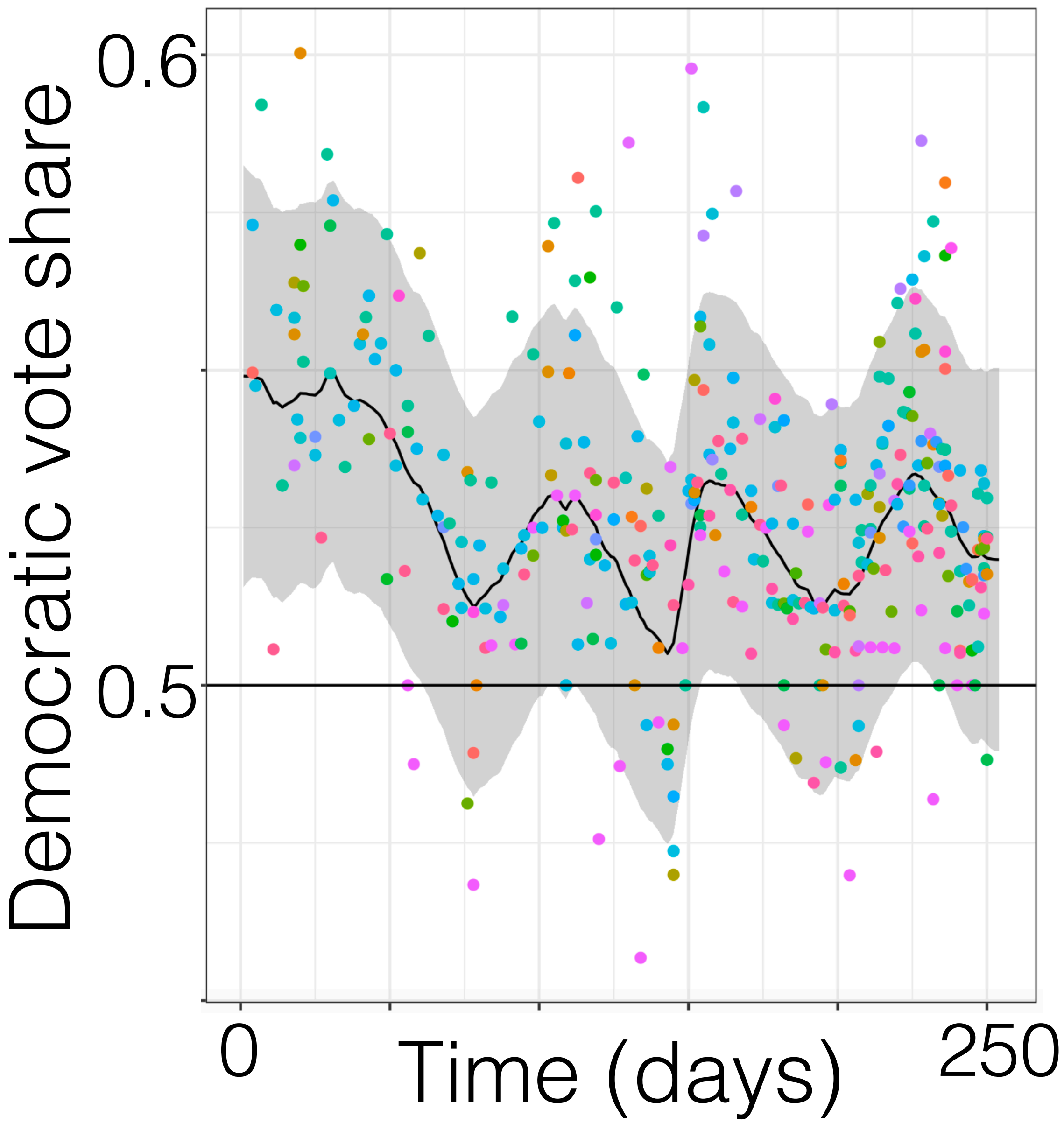

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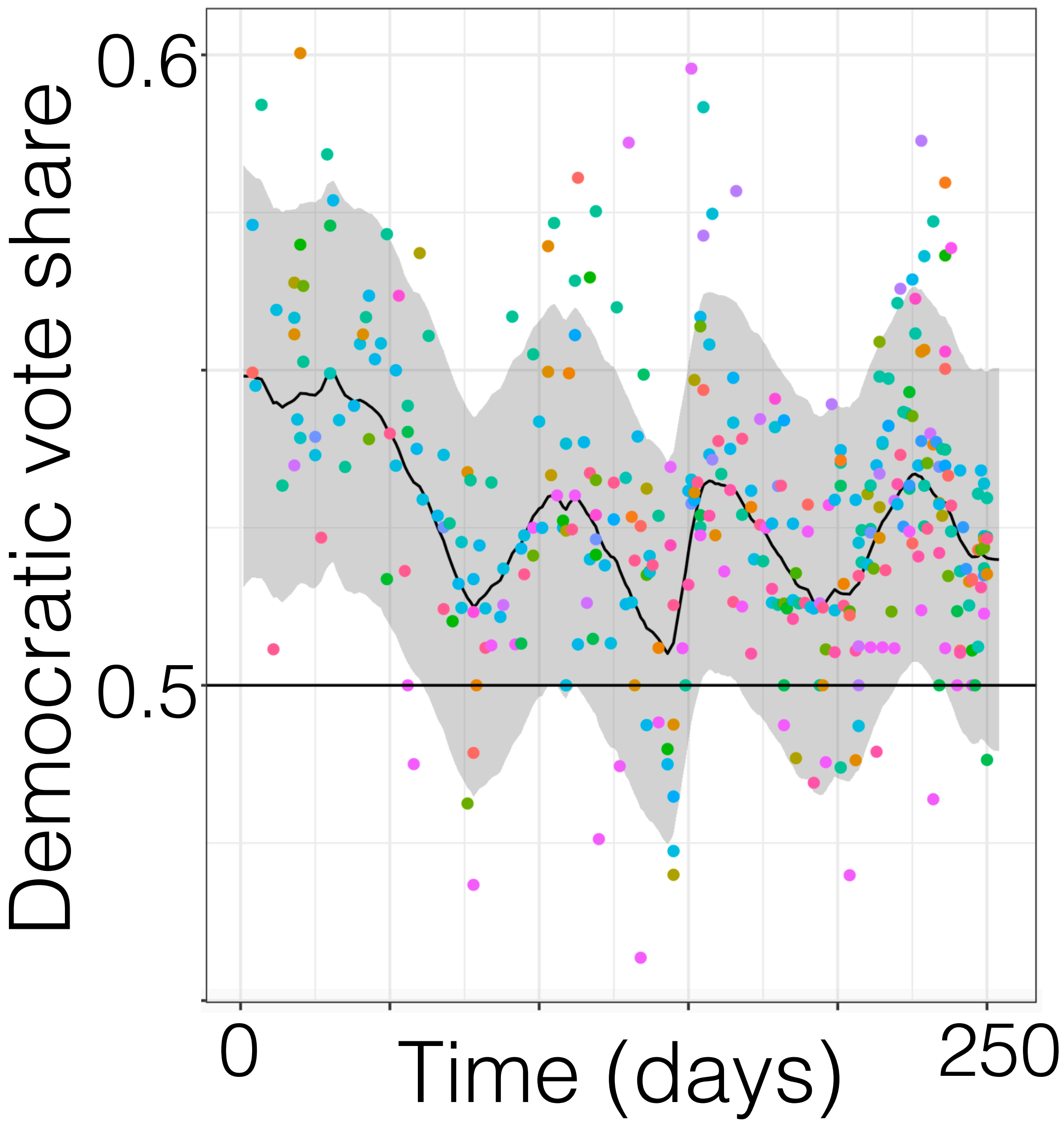

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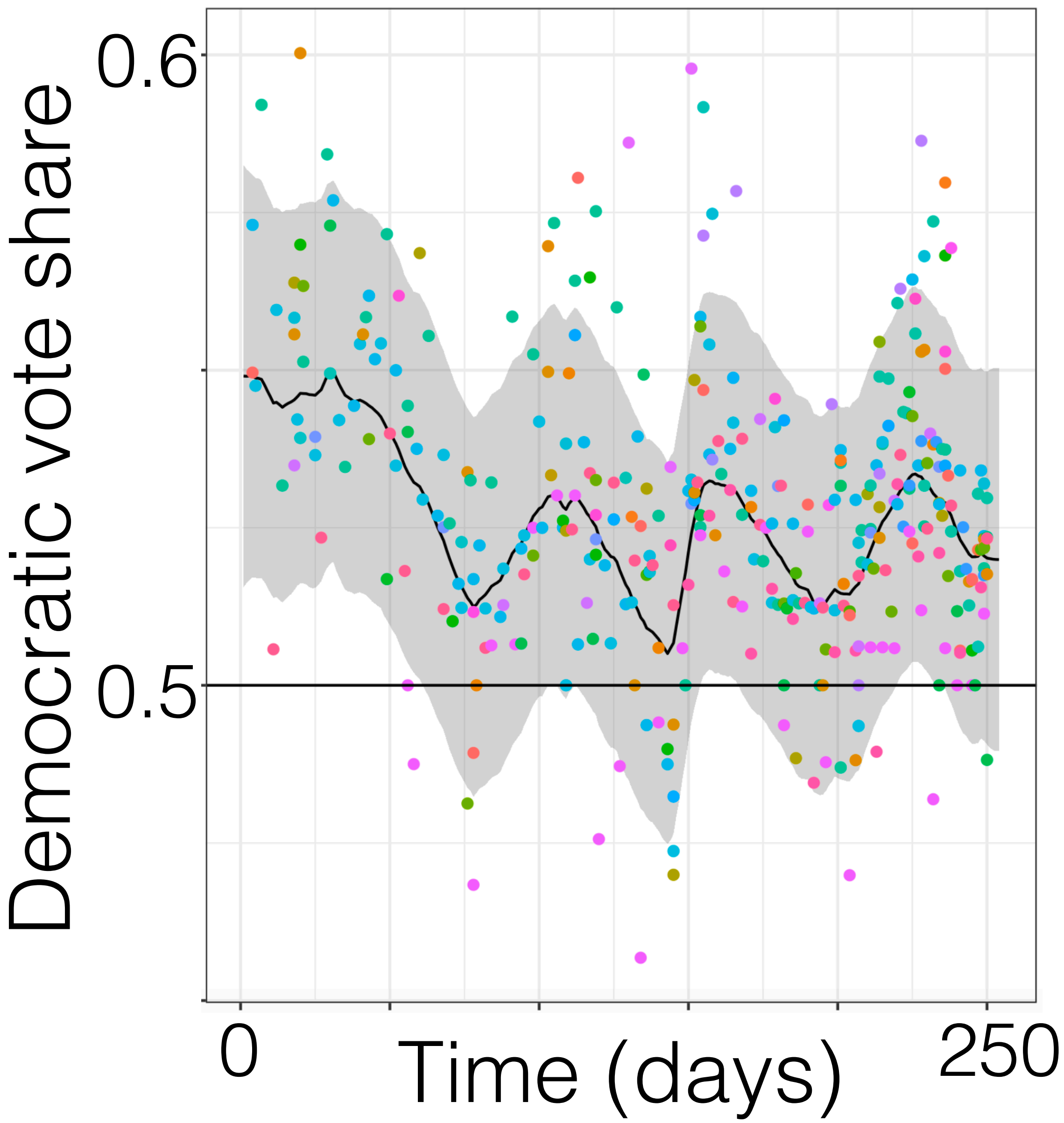

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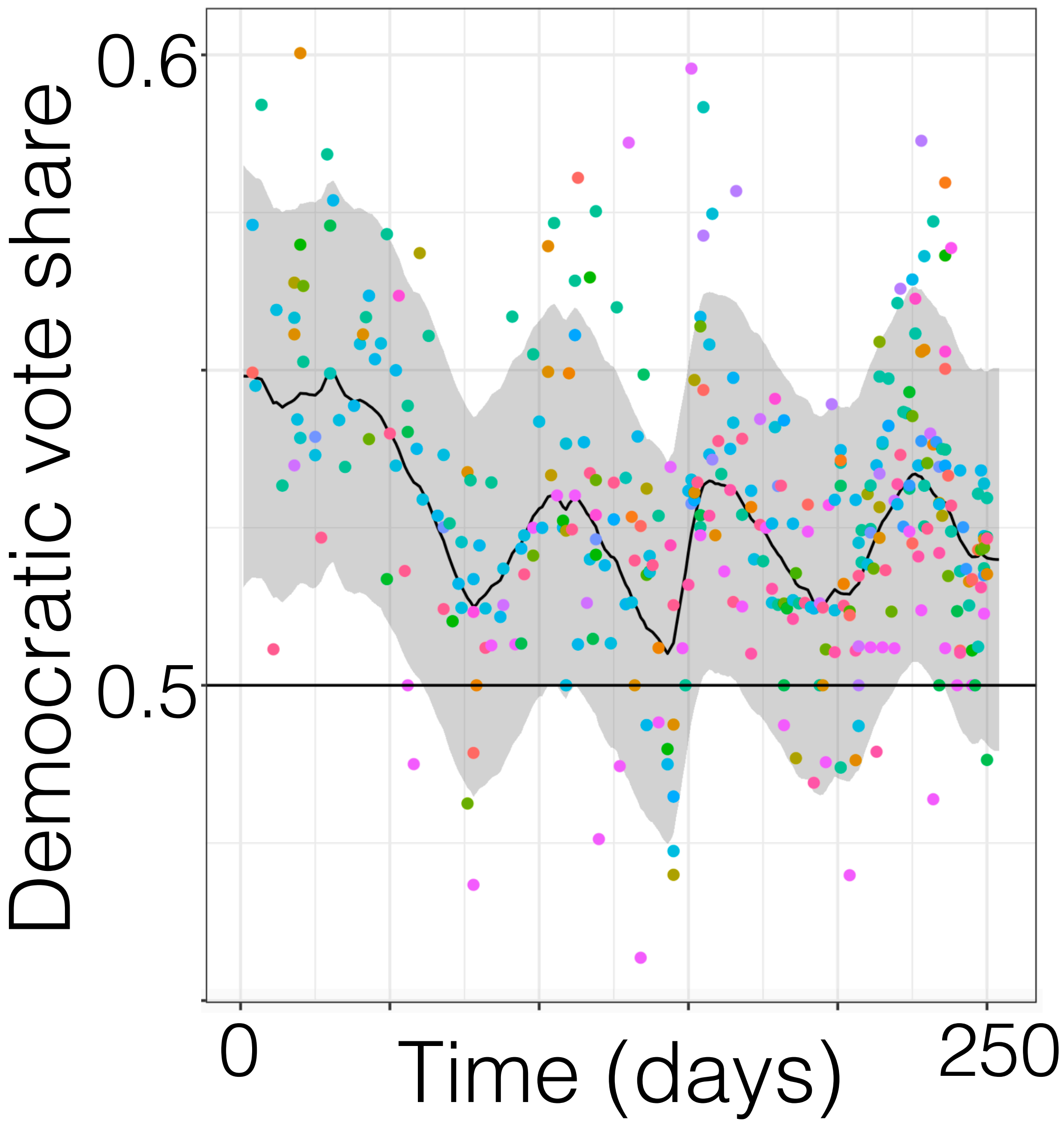

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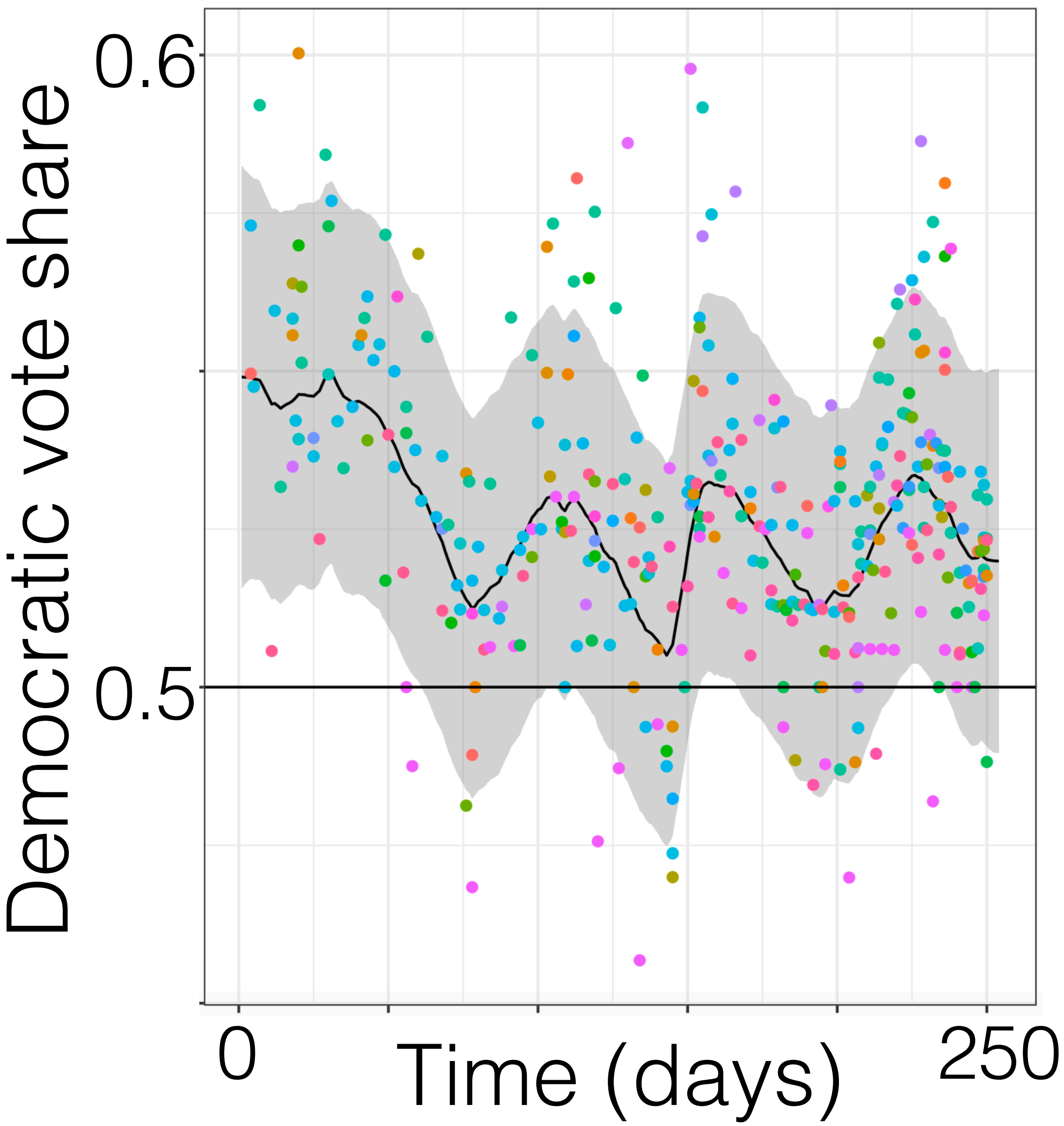

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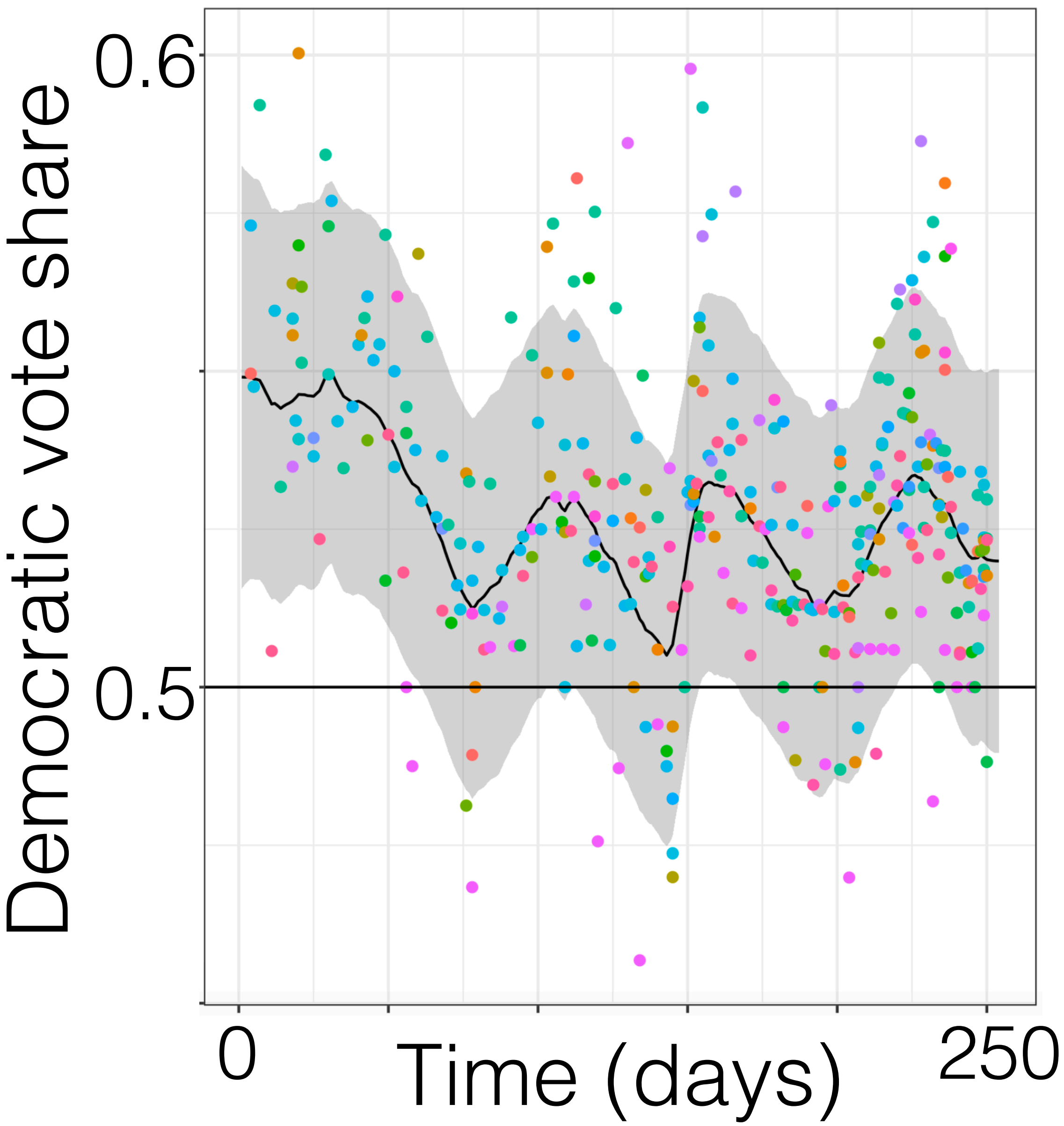

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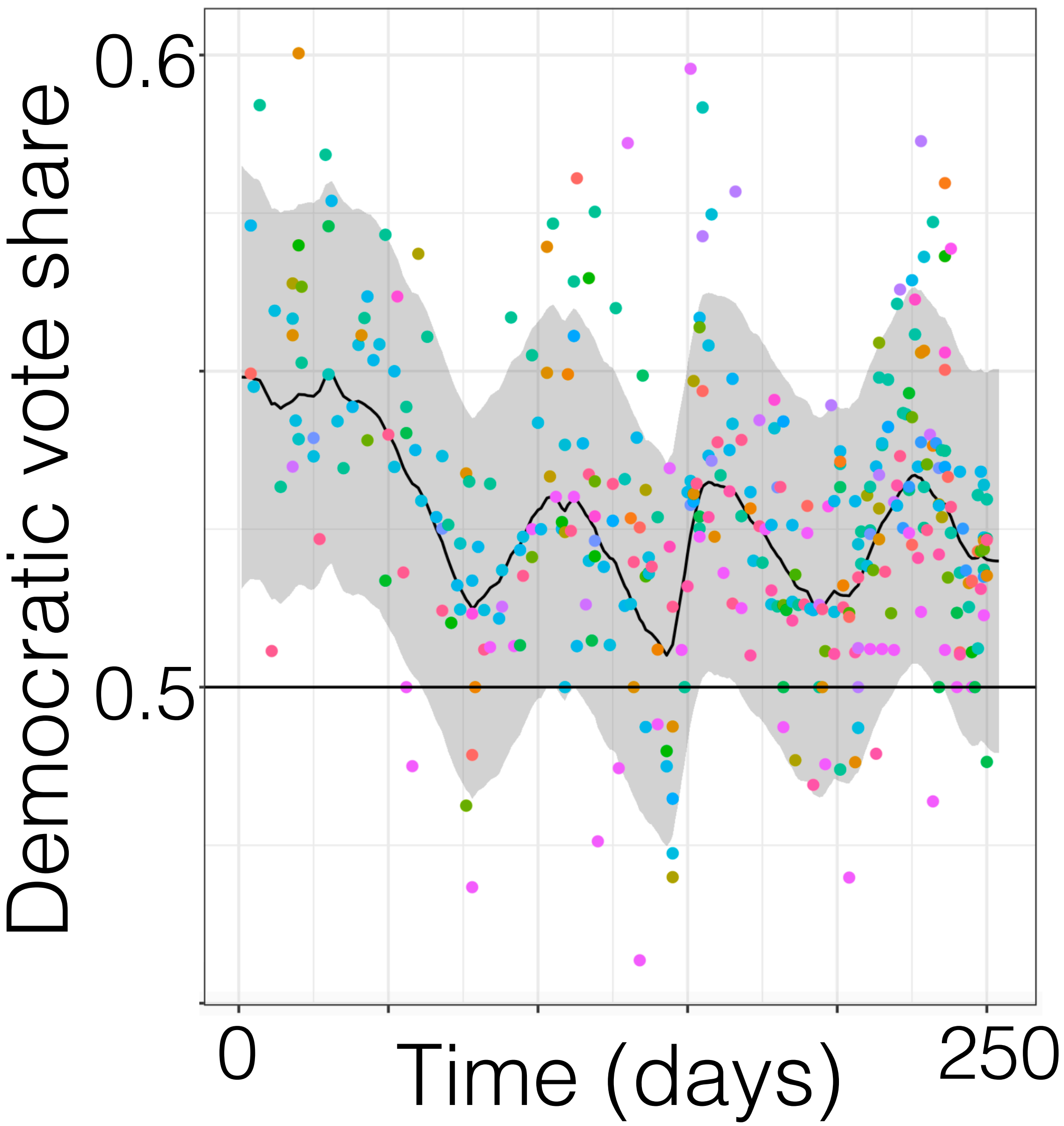

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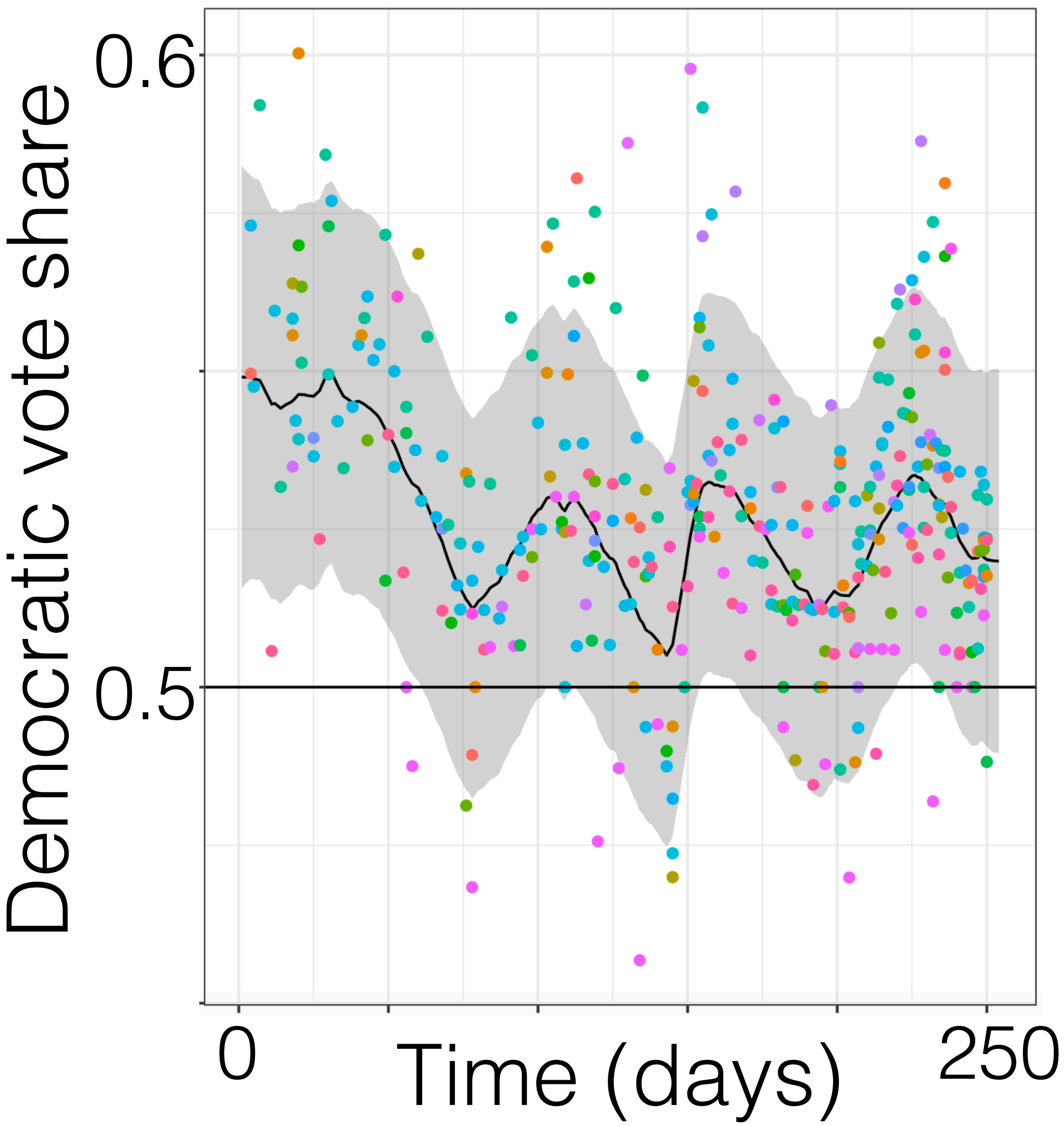

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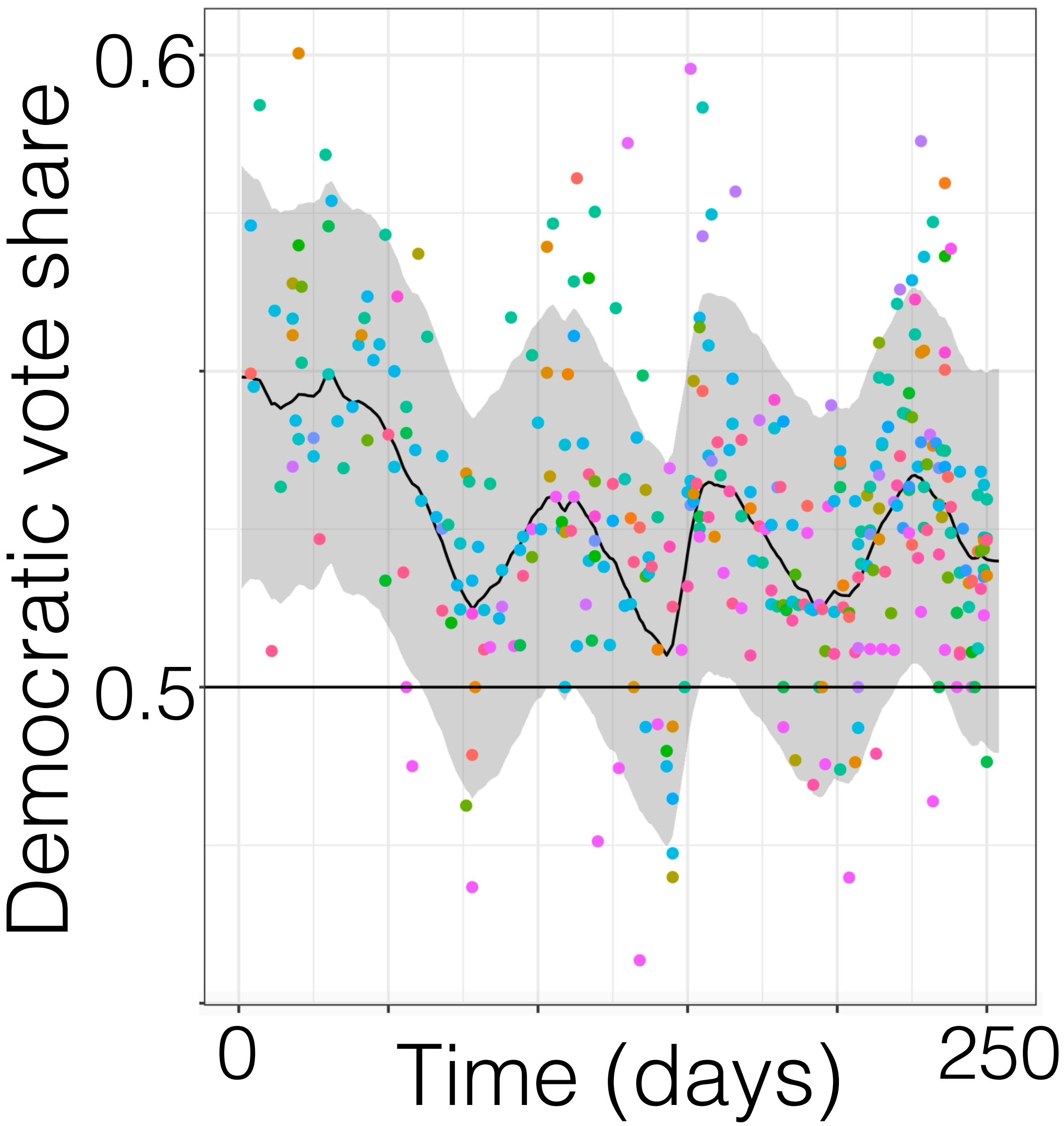

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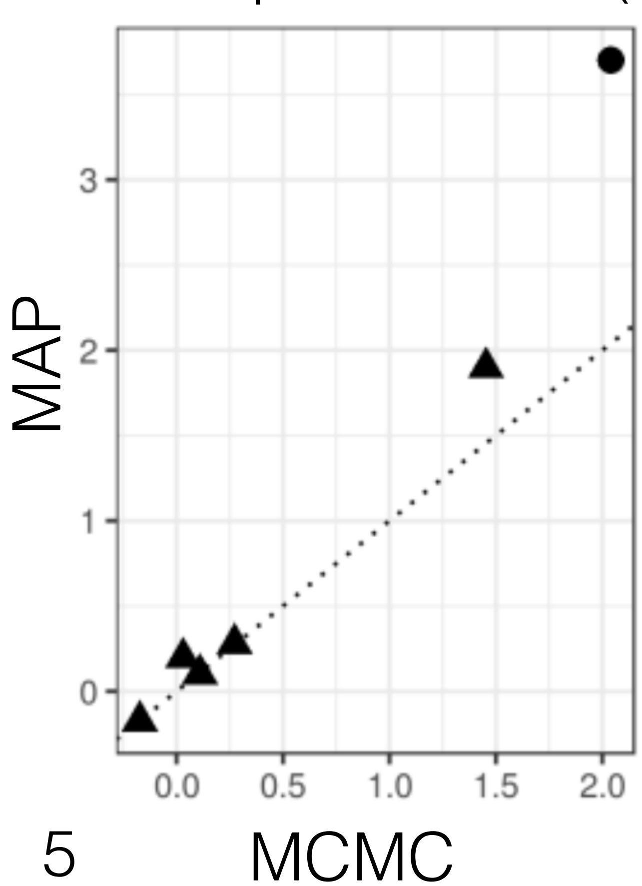

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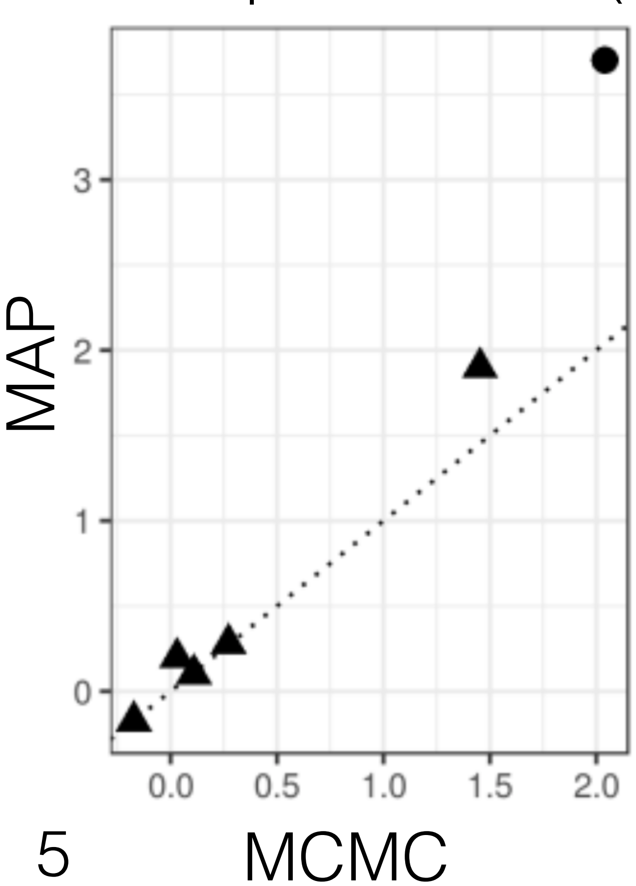

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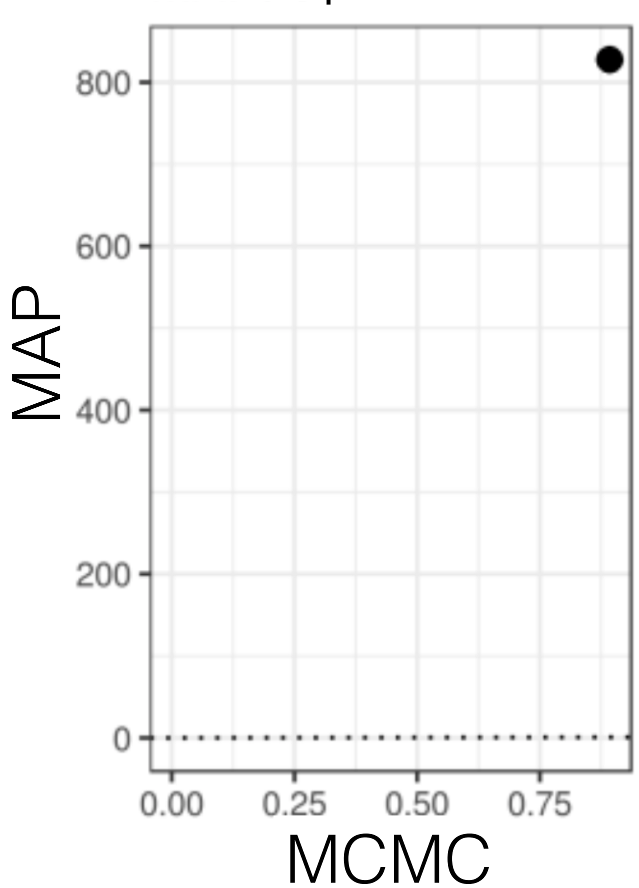

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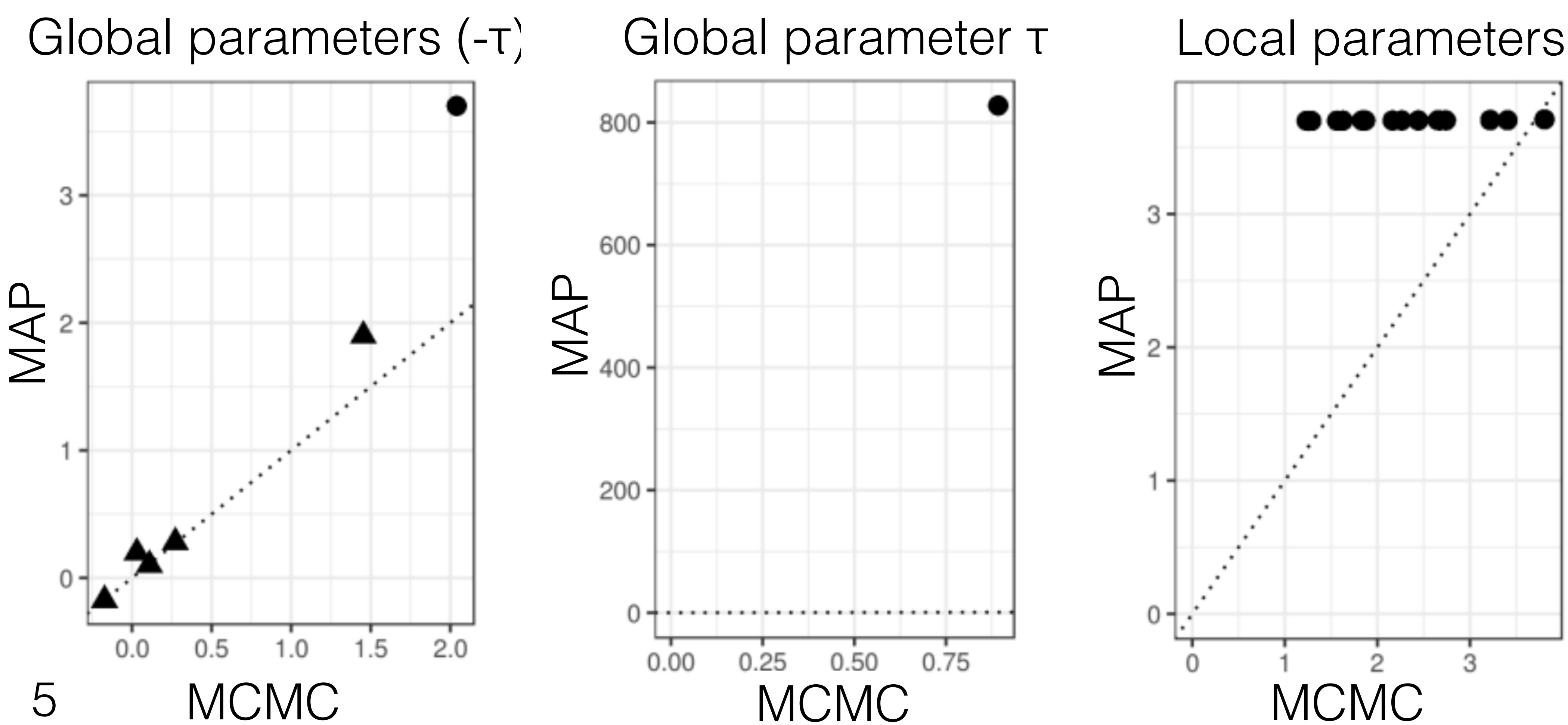

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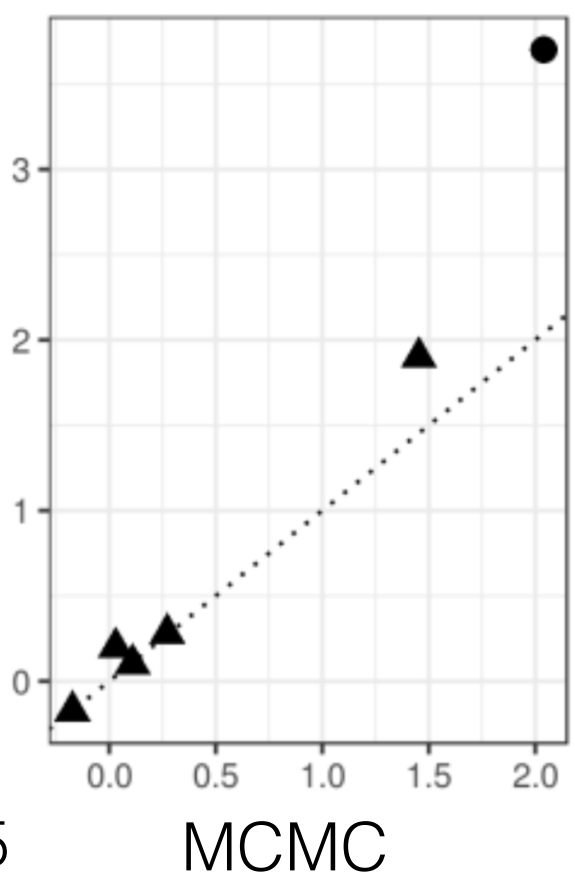

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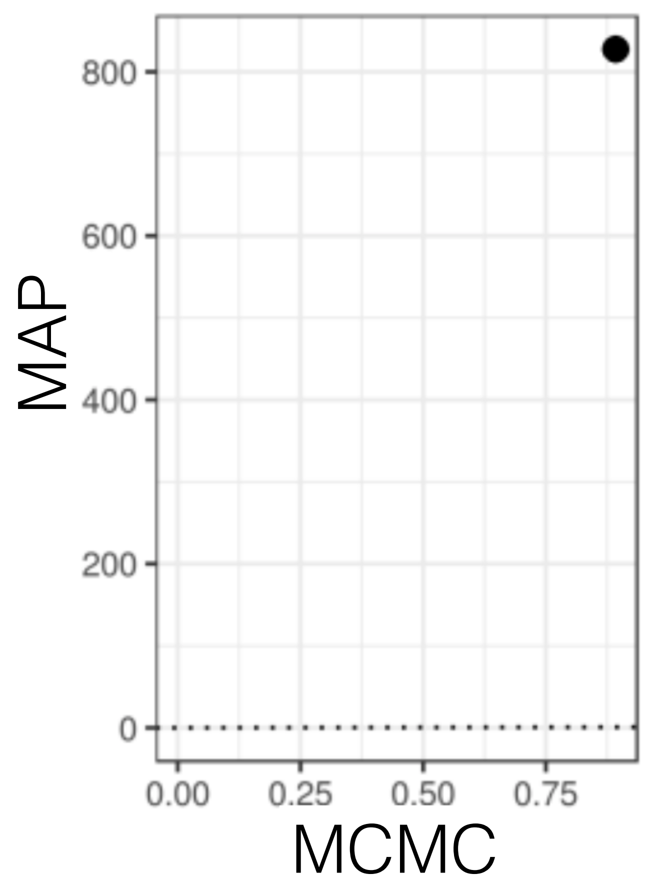
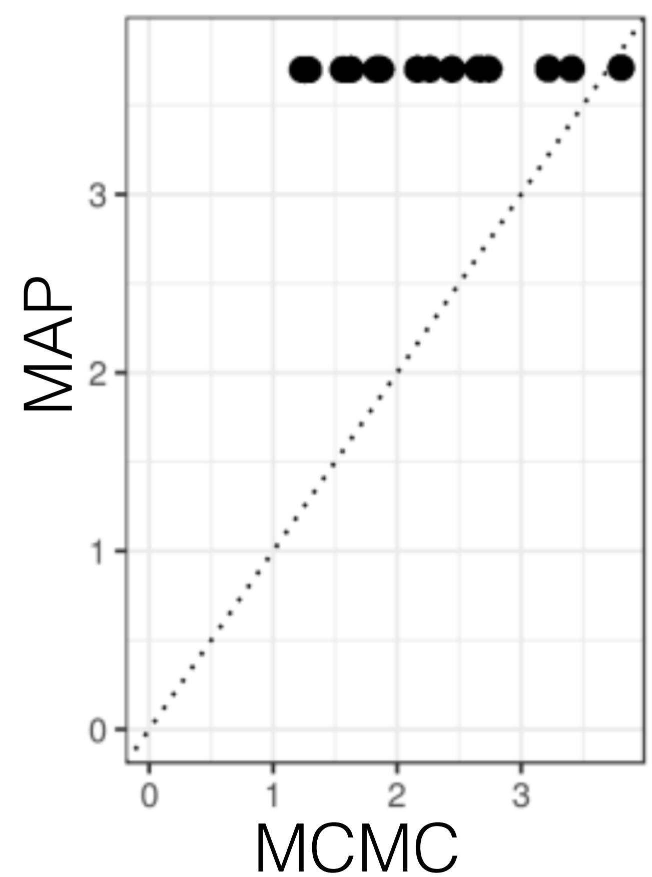


Local parameters



- MAP: 12 s
- MCMC (5K samples): 5.85 h

[Giordano, Broderick, Jordan 2018]

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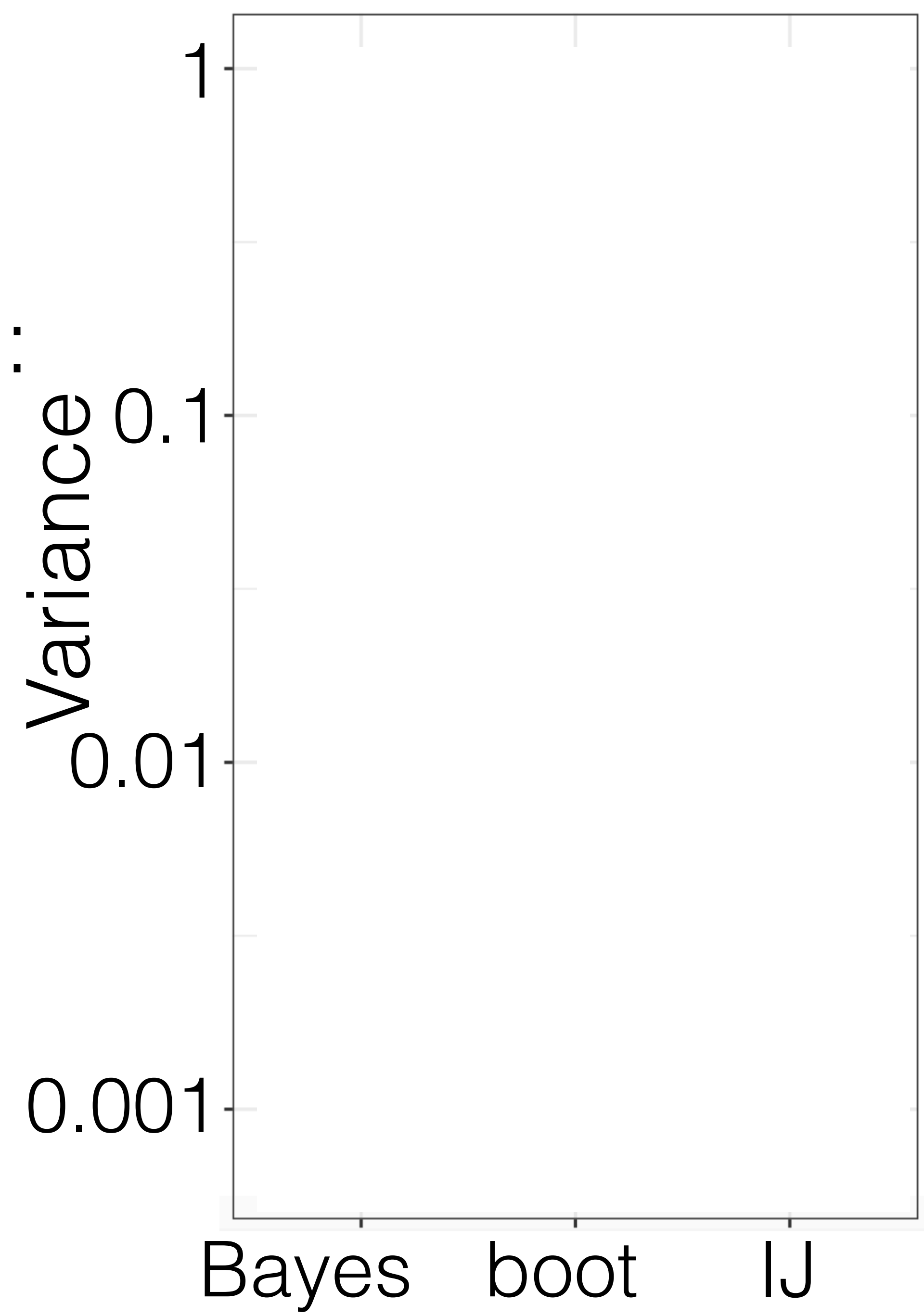

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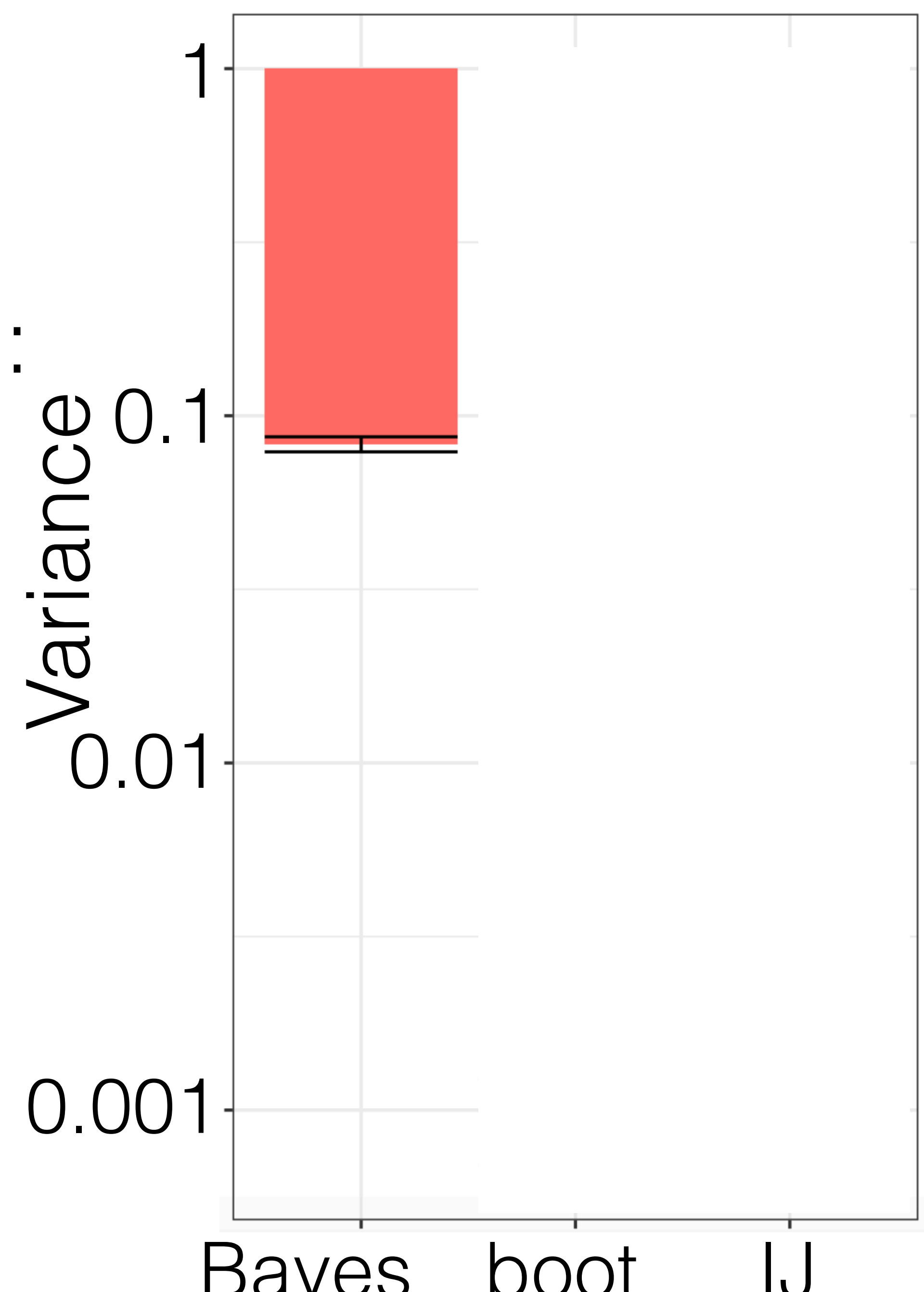

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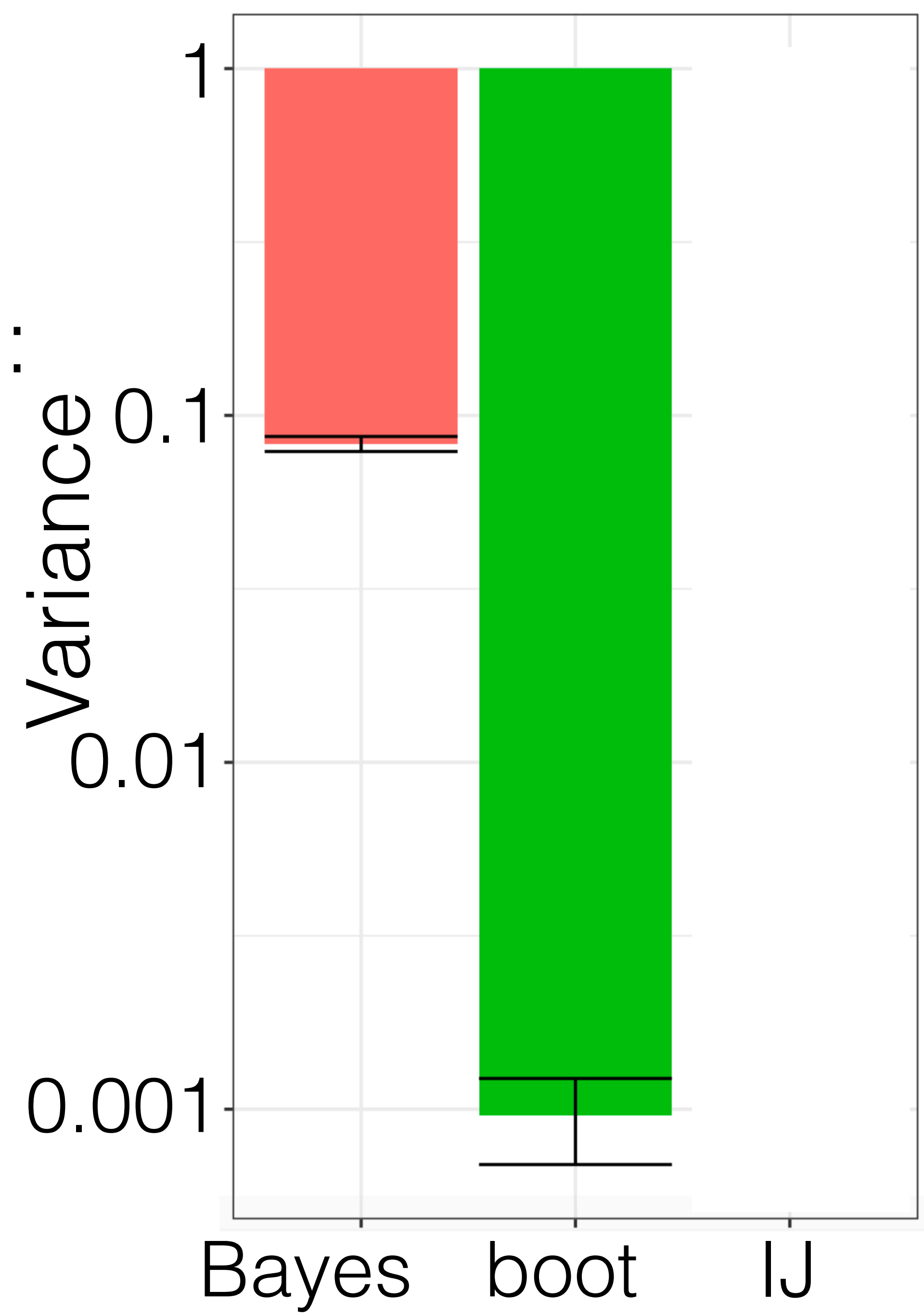

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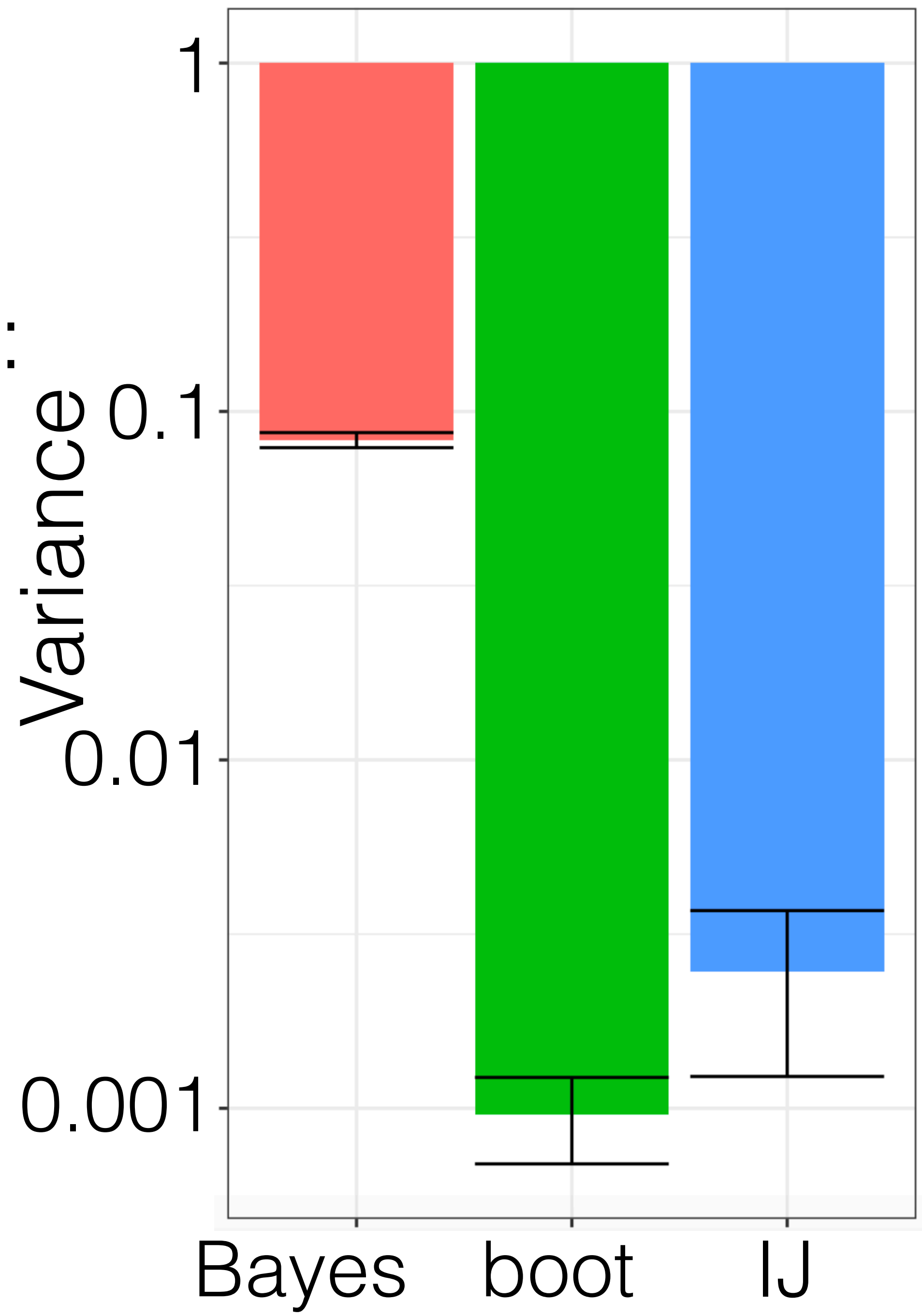

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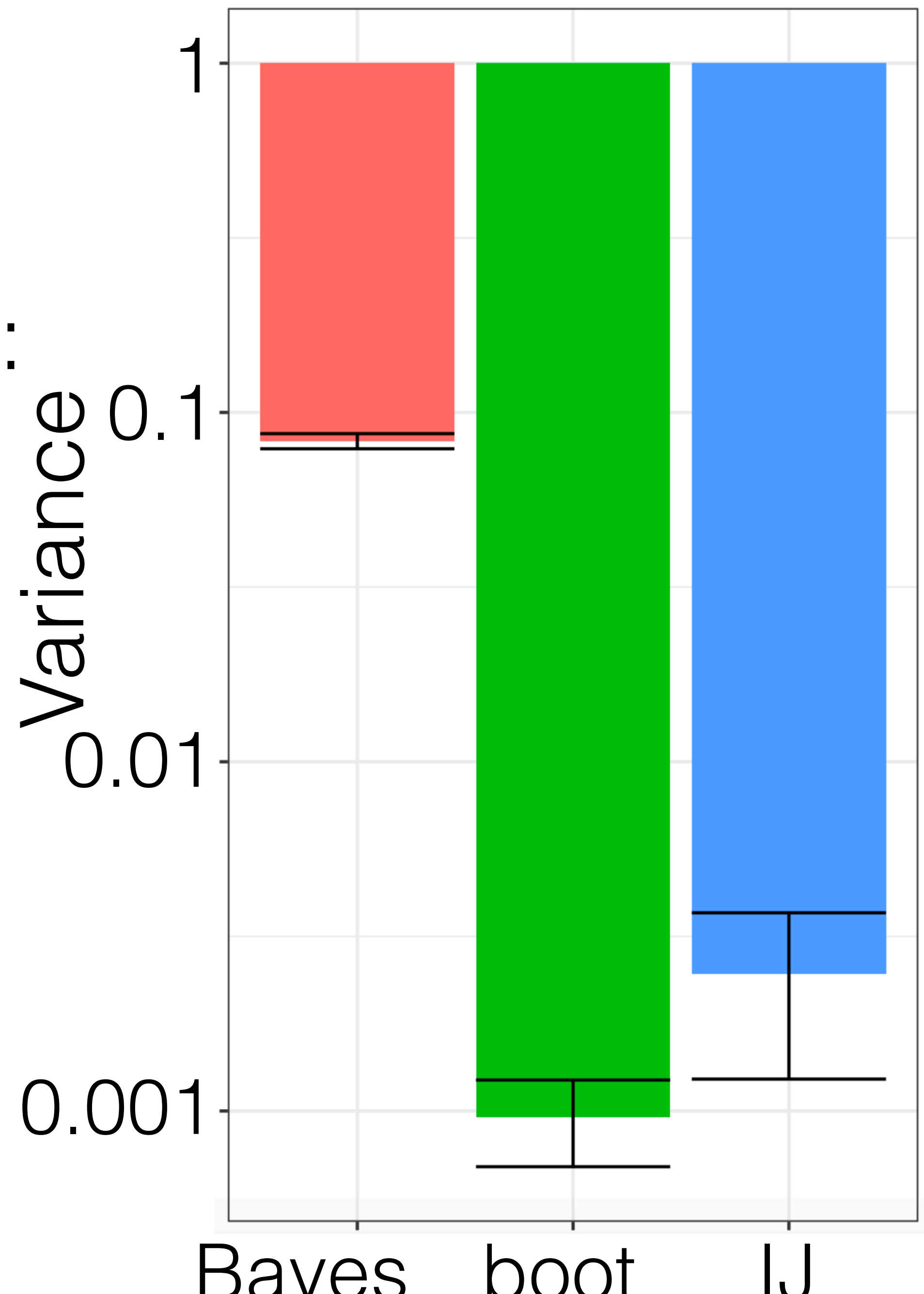

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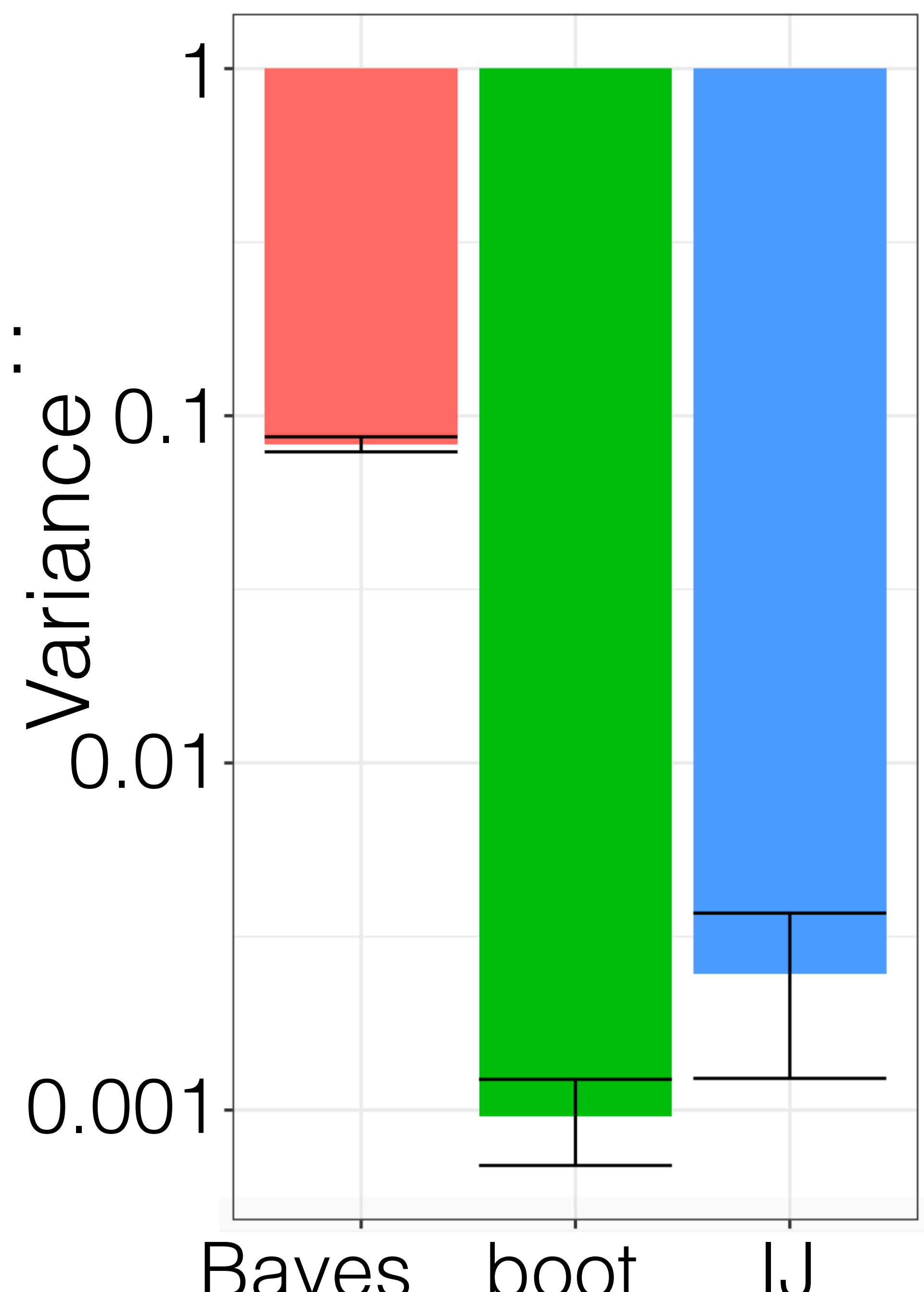

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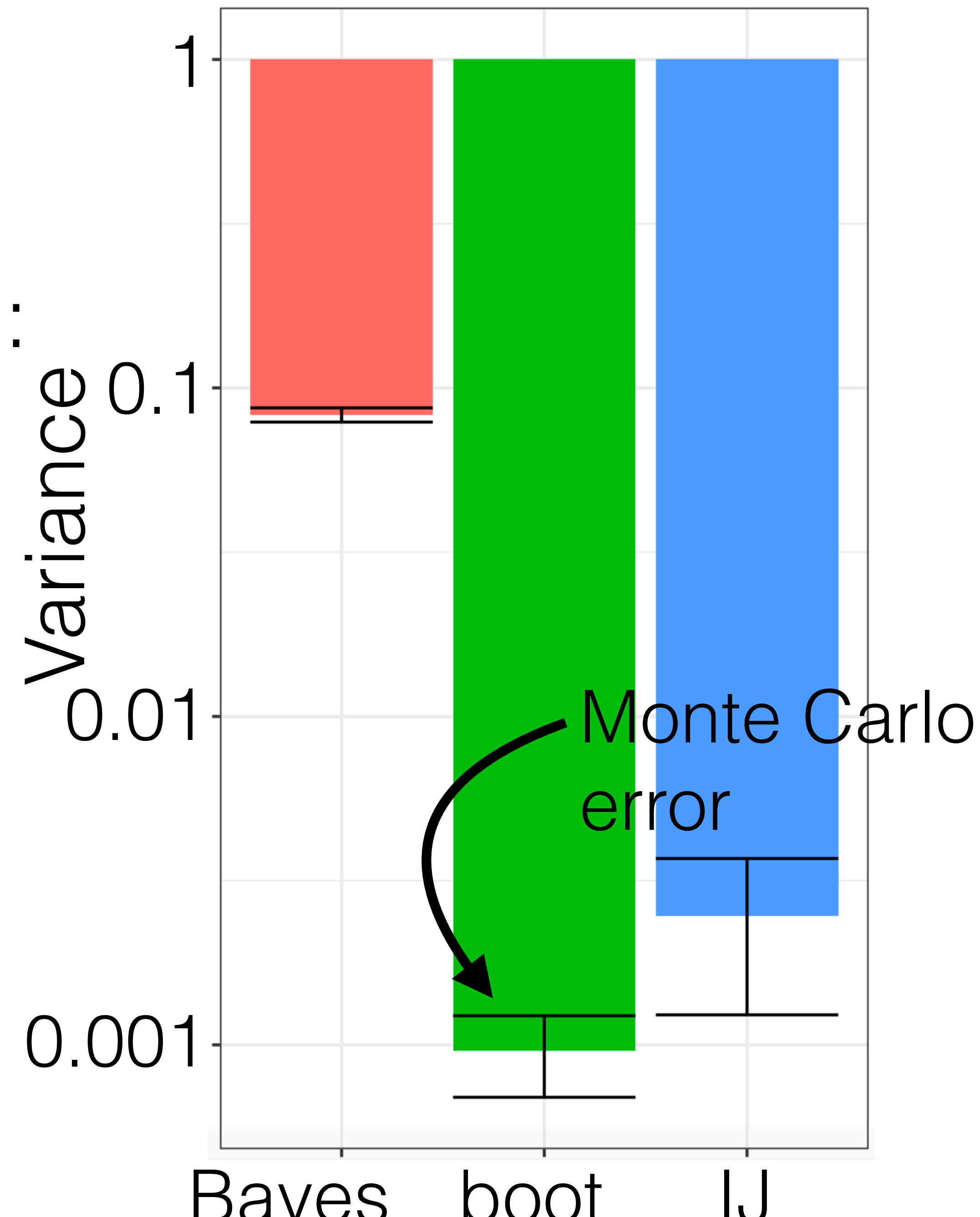

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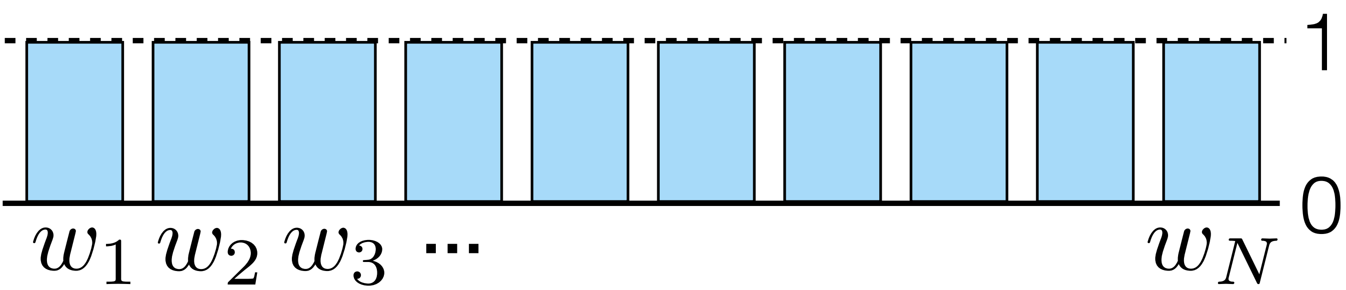

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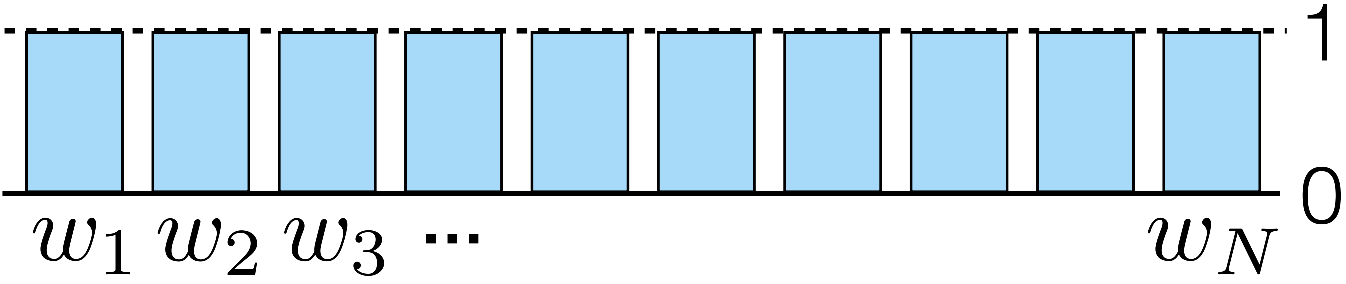

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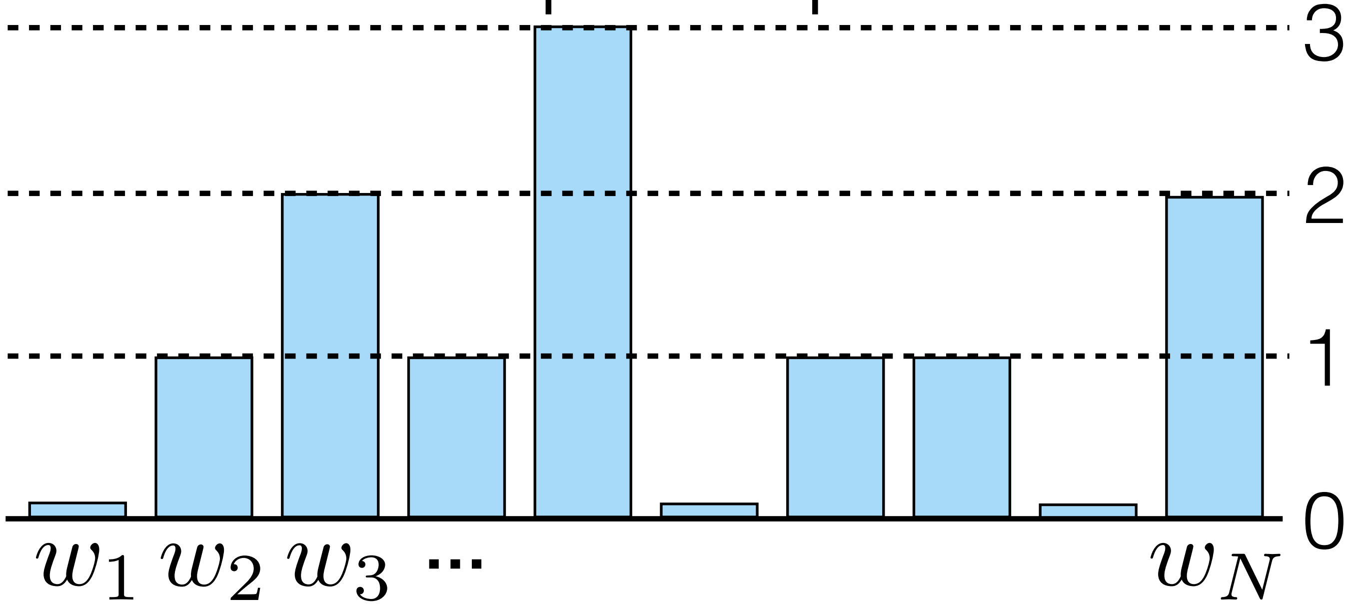

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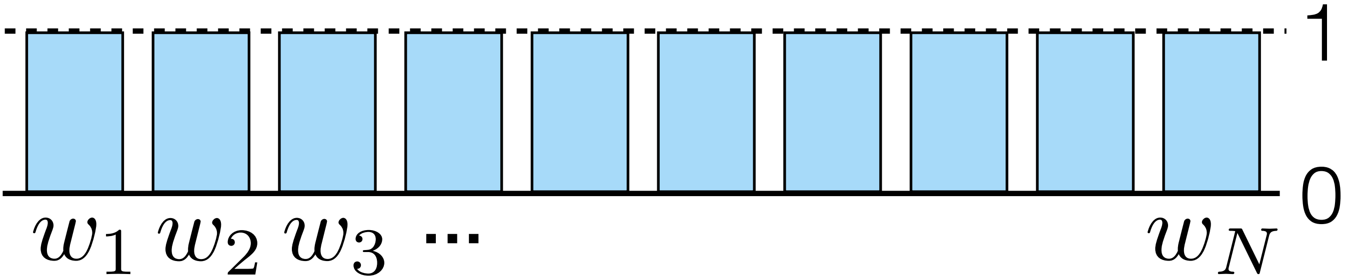

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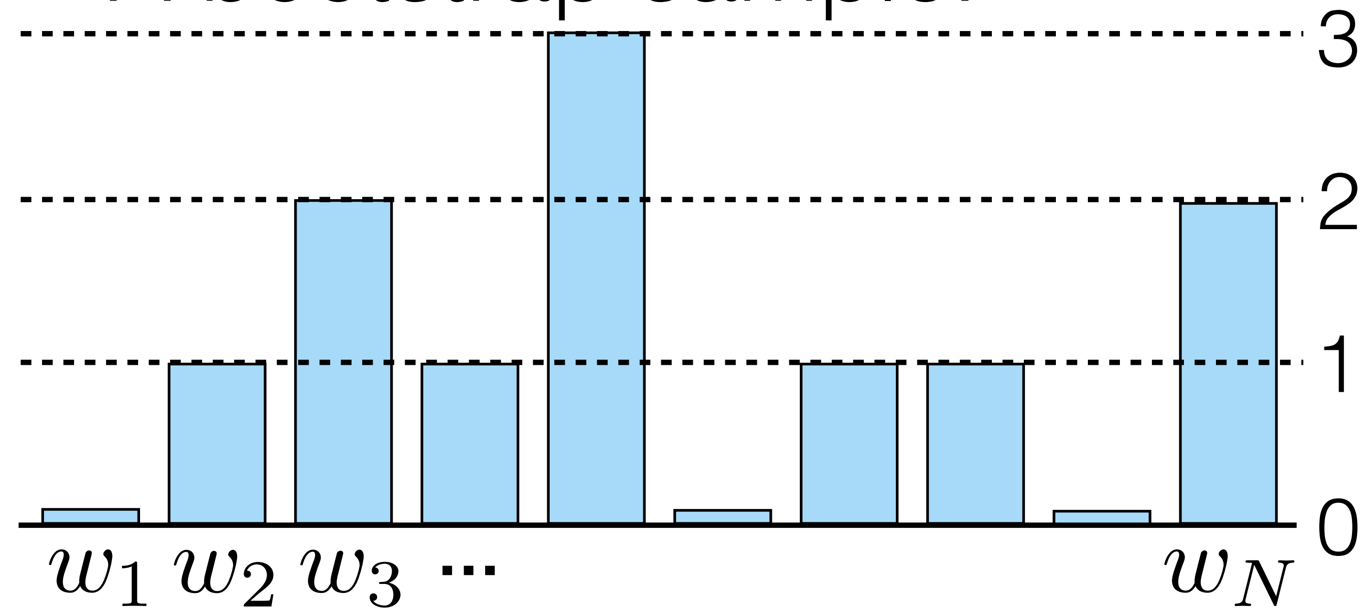

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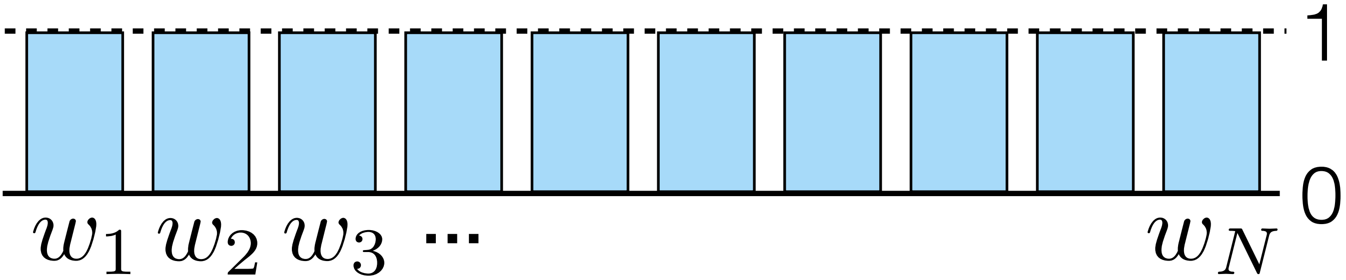

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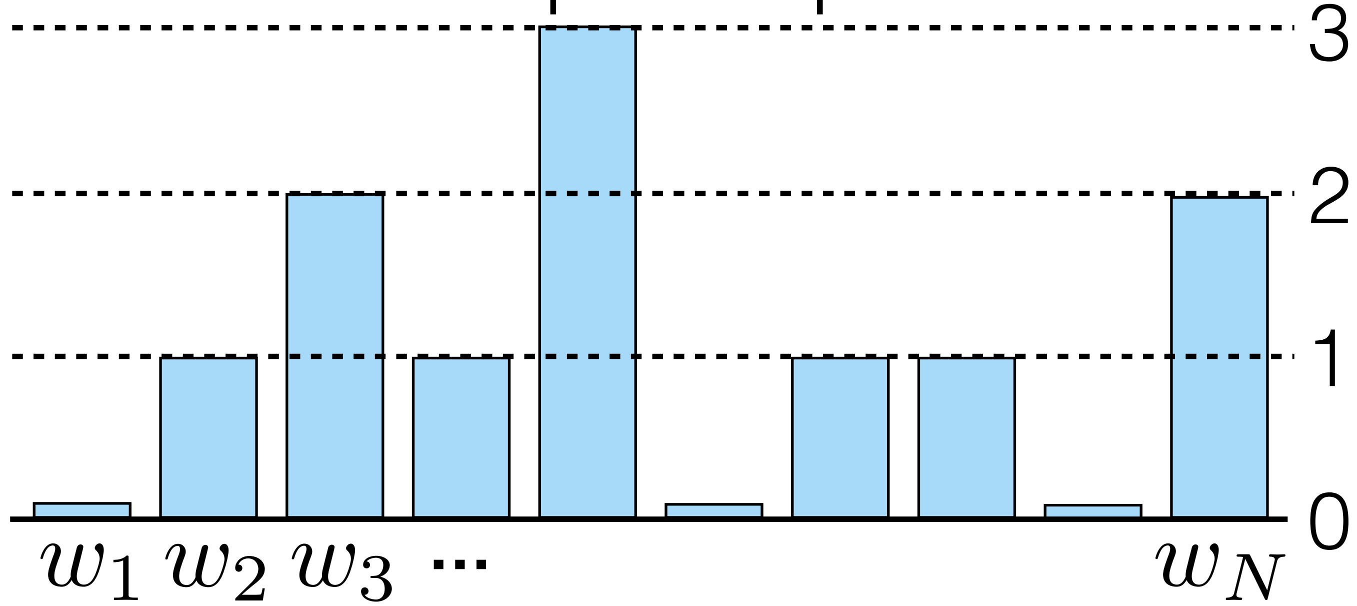

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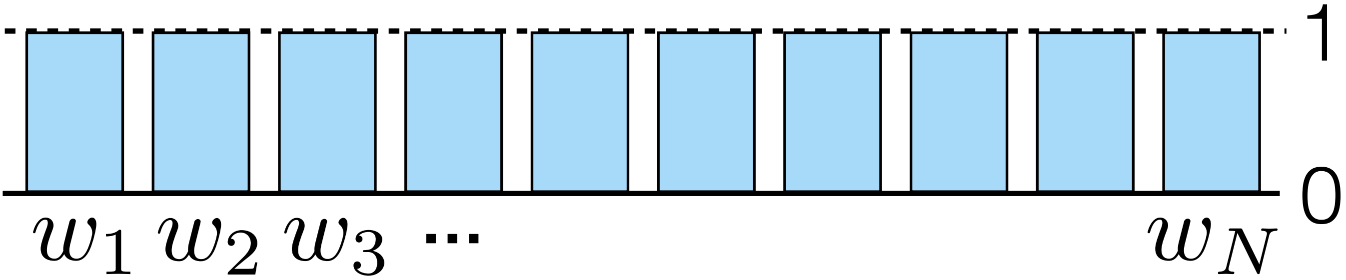

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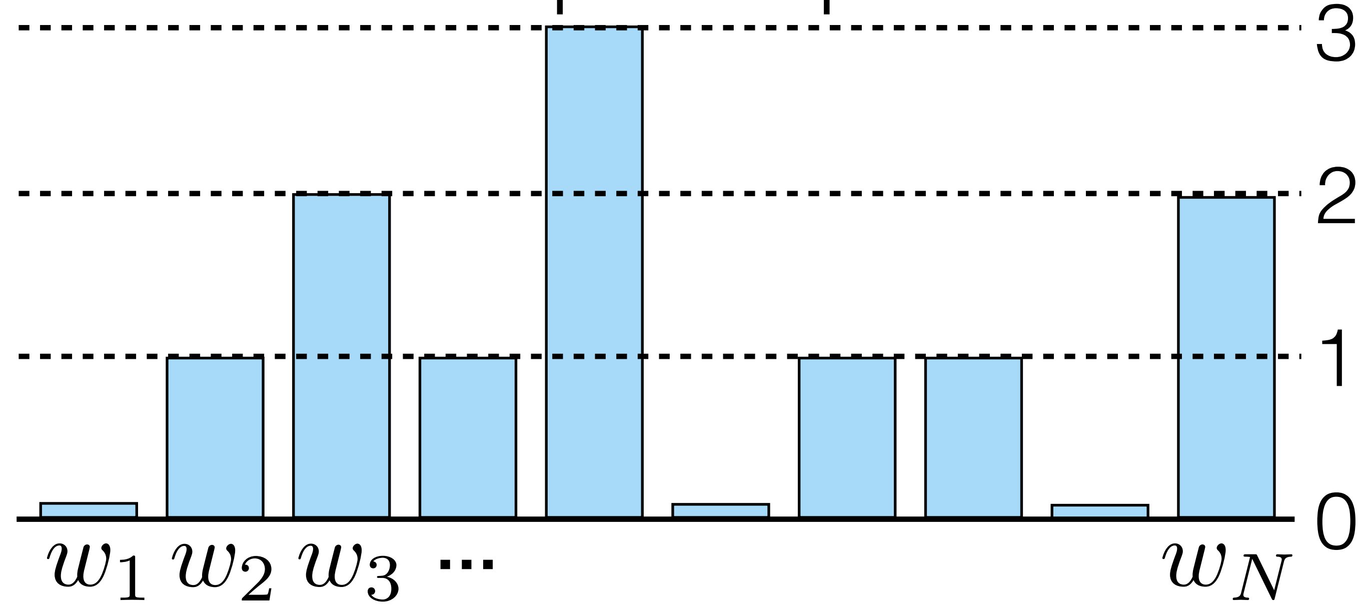

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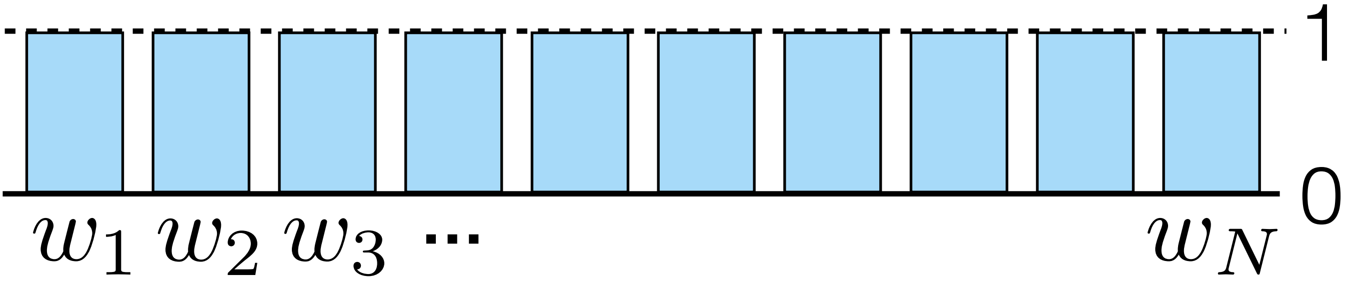

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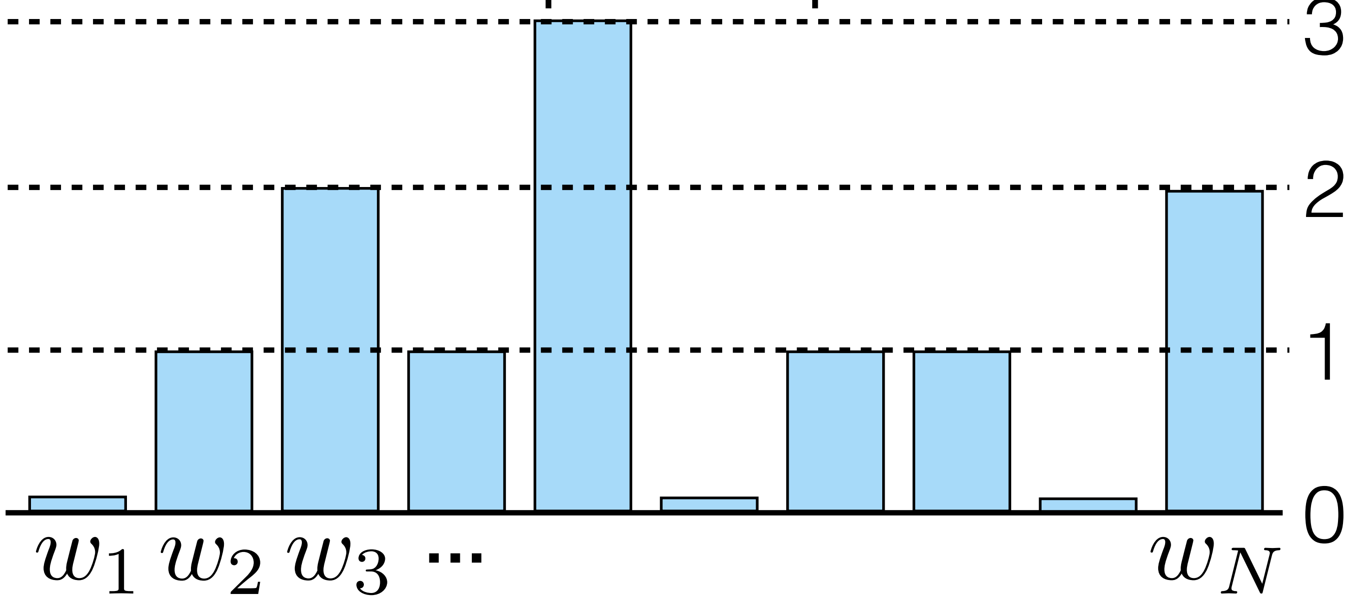

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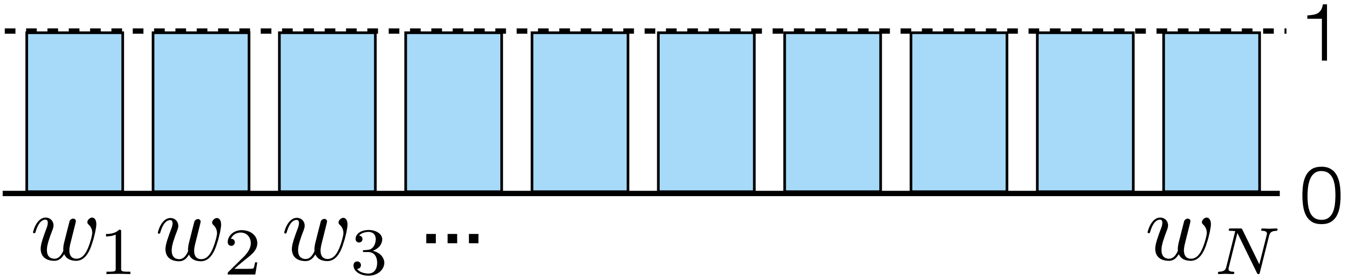

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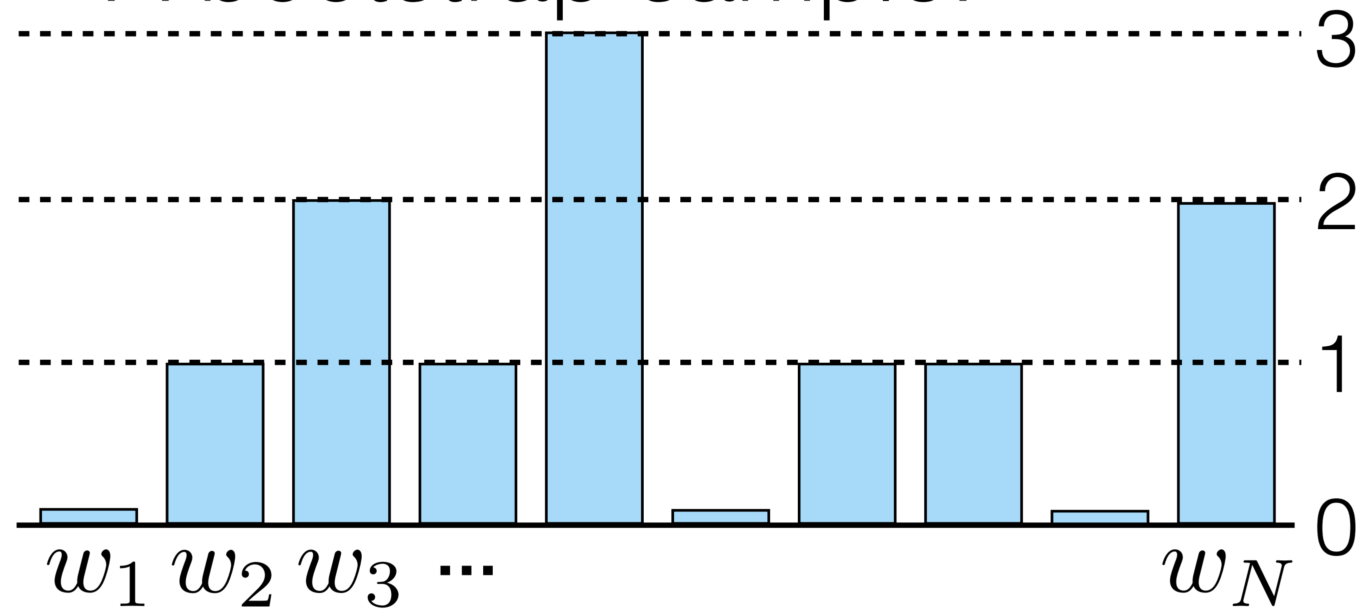

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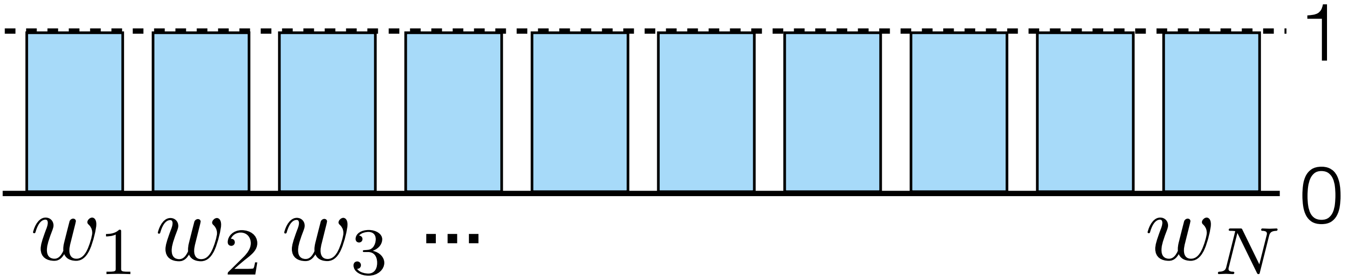

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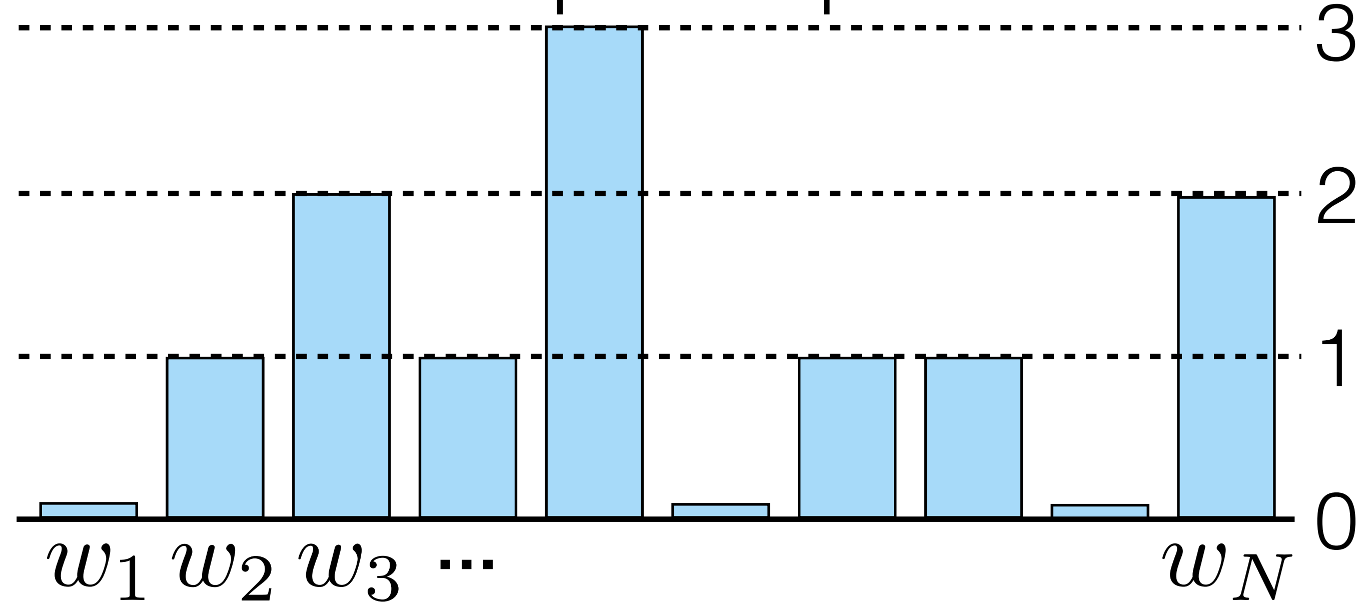

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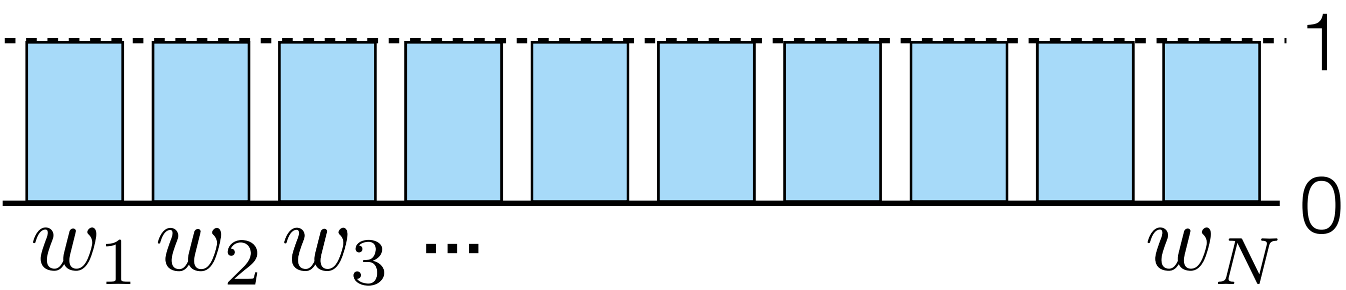

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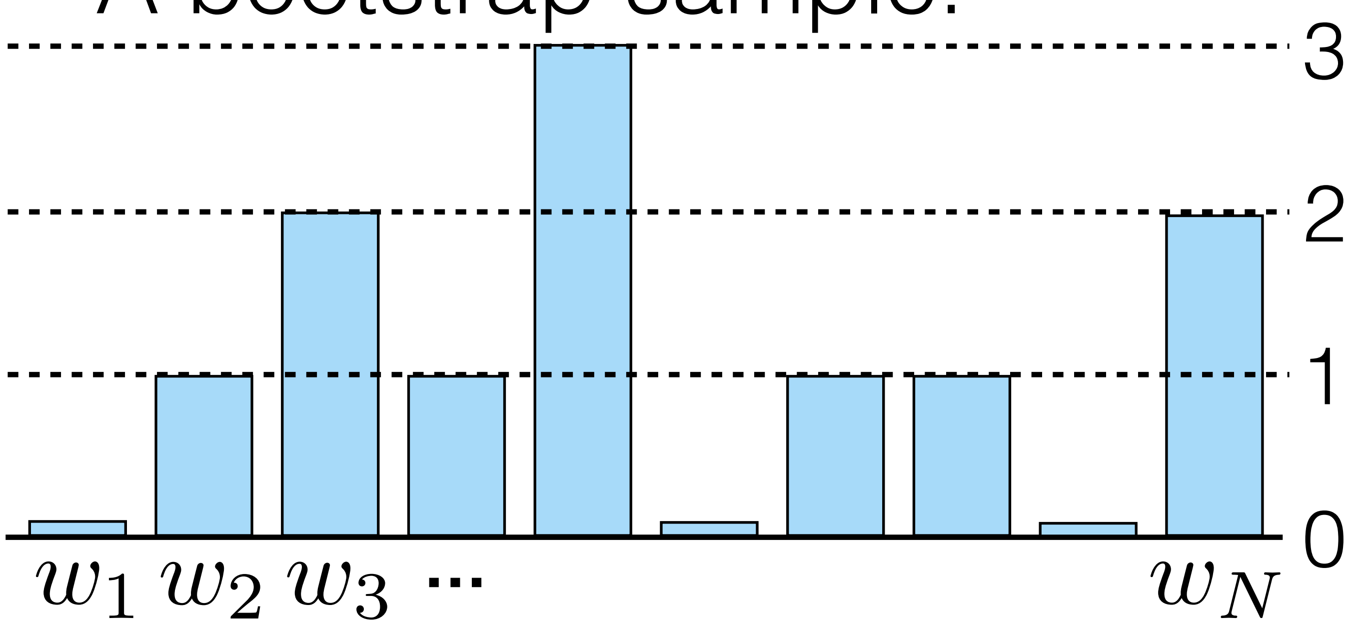

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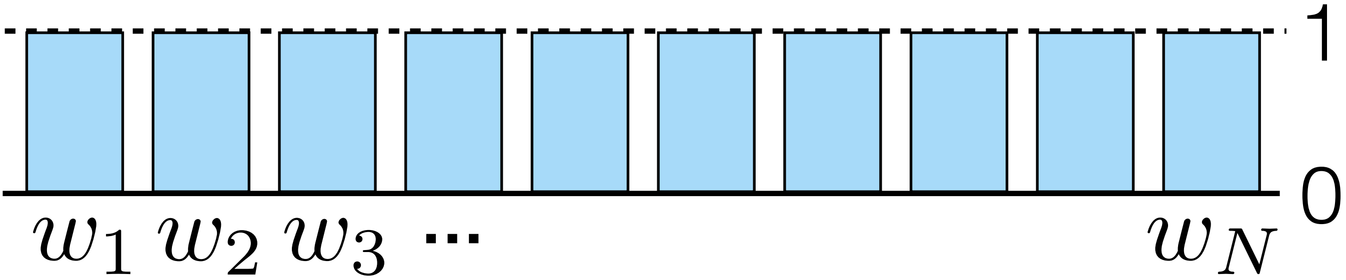

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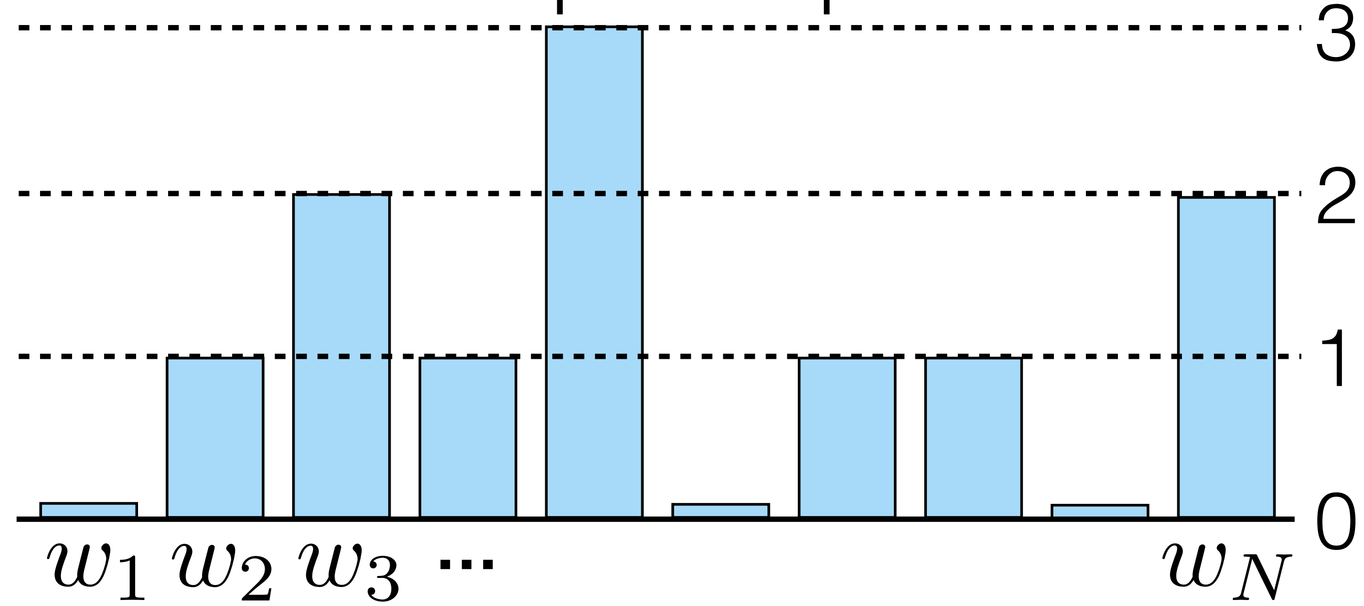

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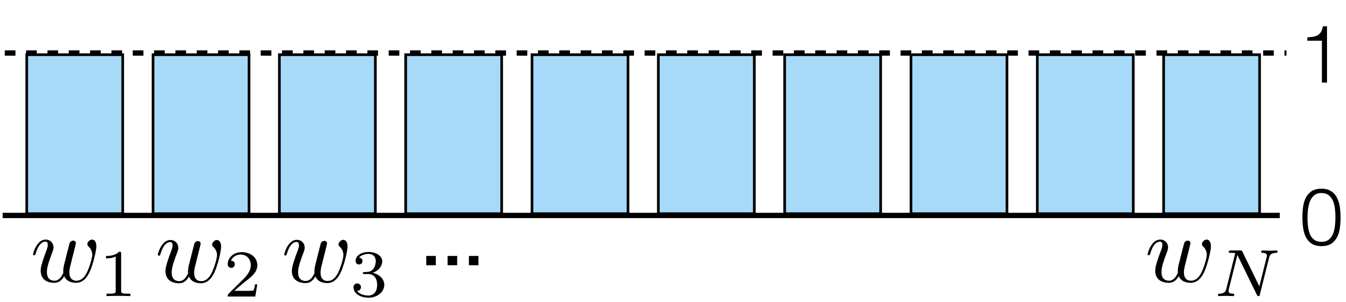

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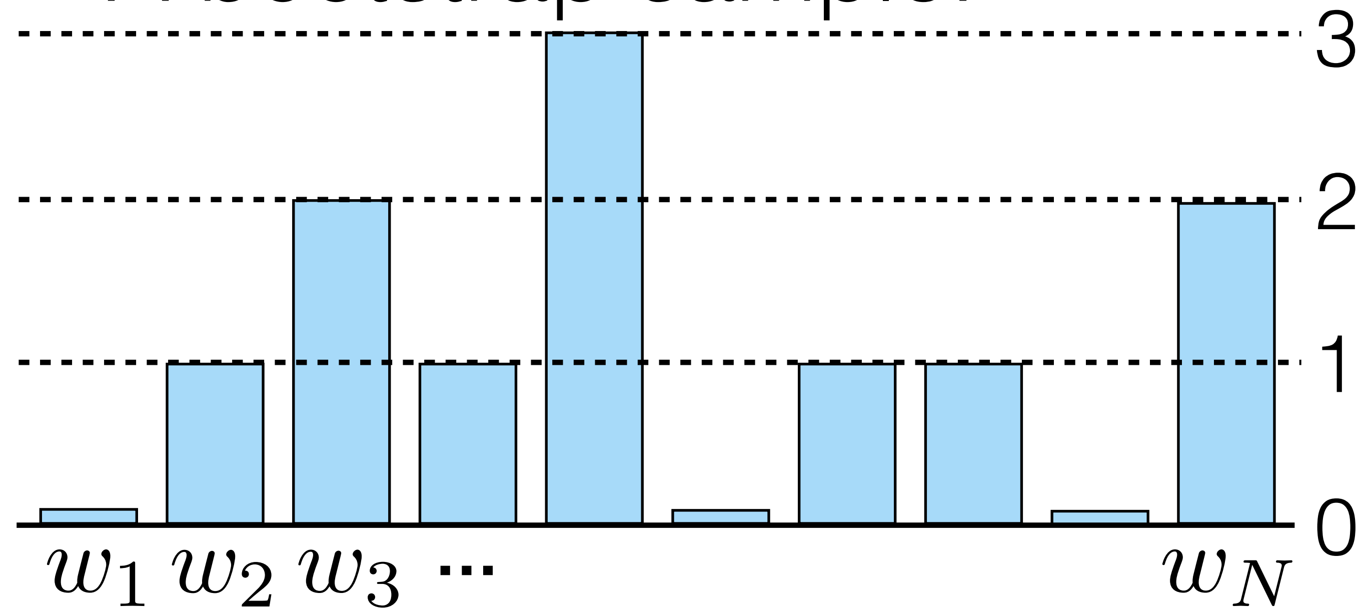

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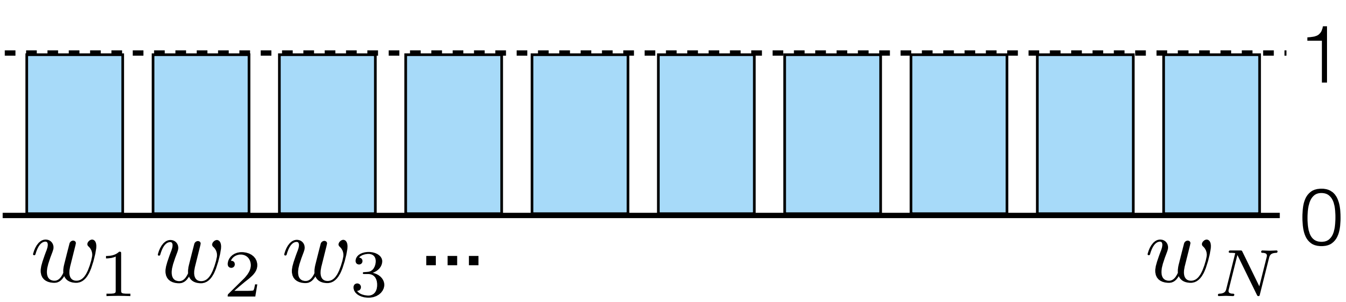

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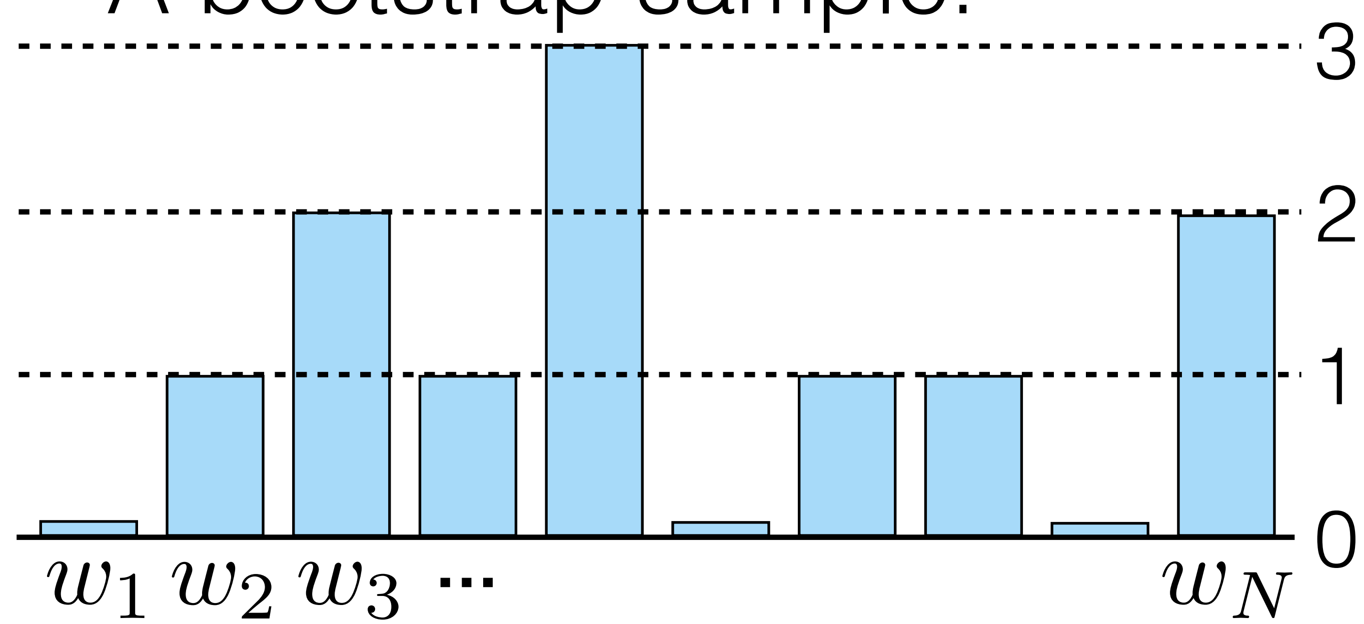

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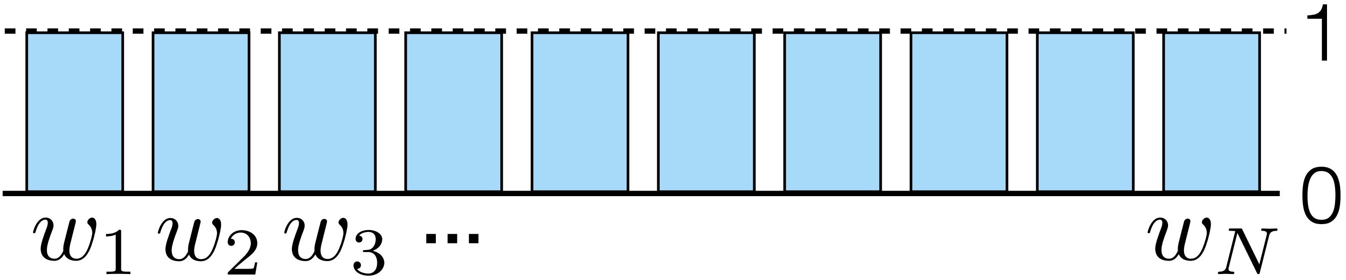

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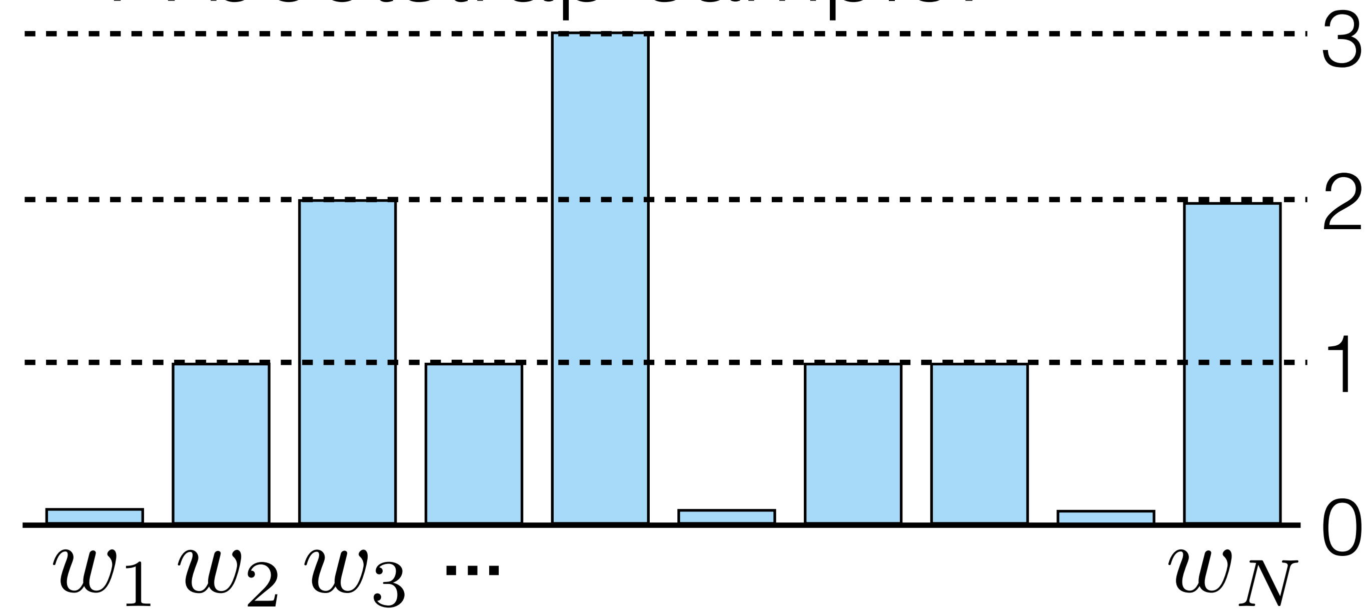

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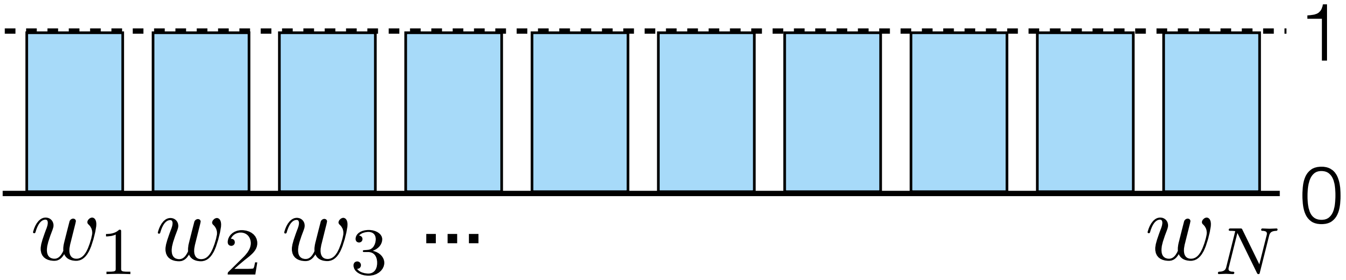

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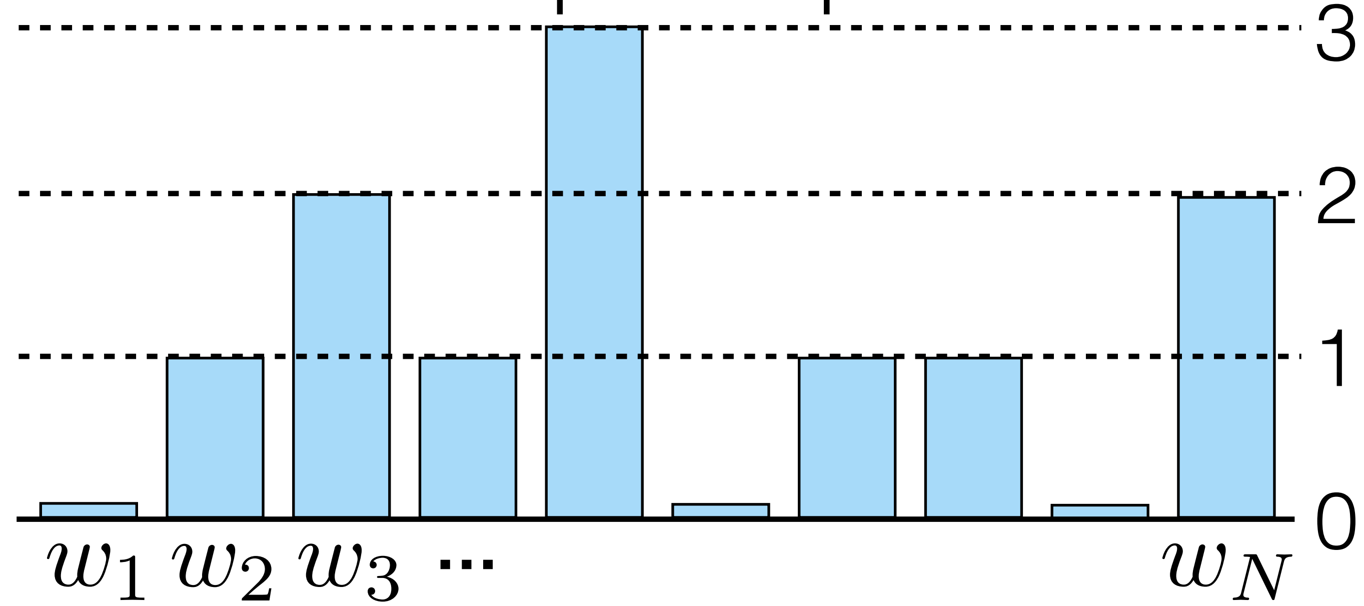

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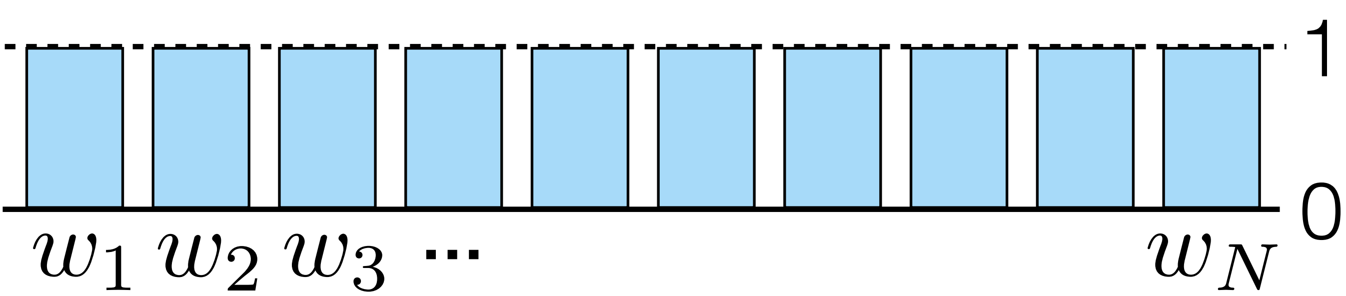

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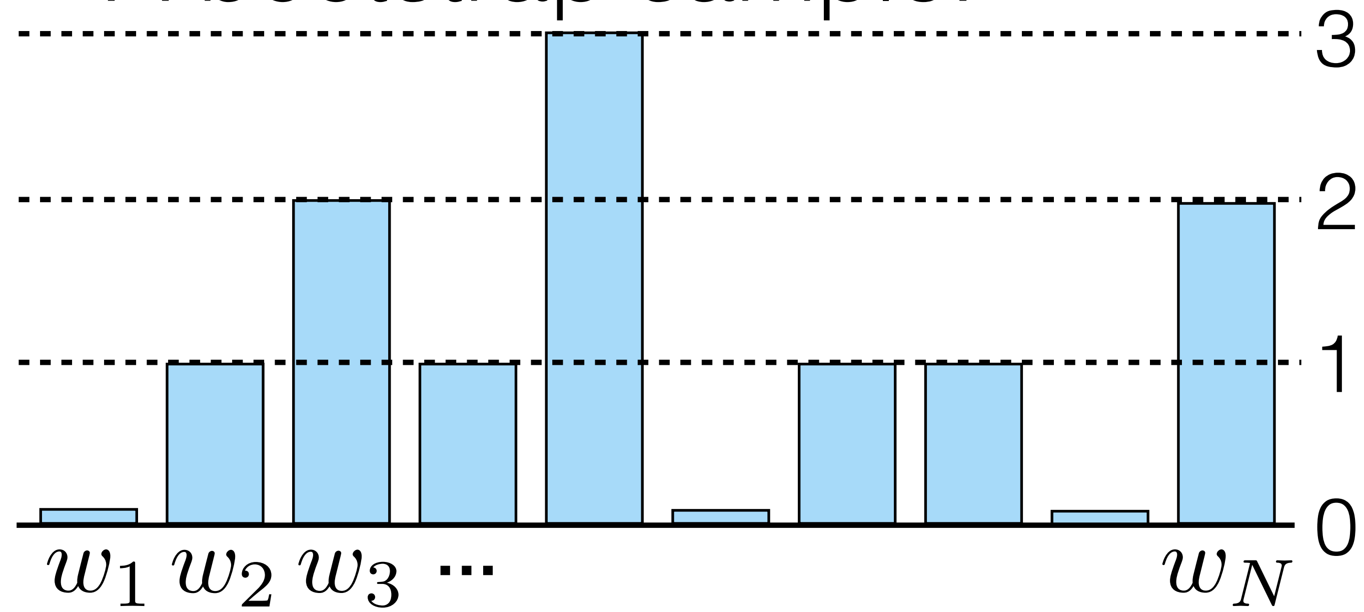

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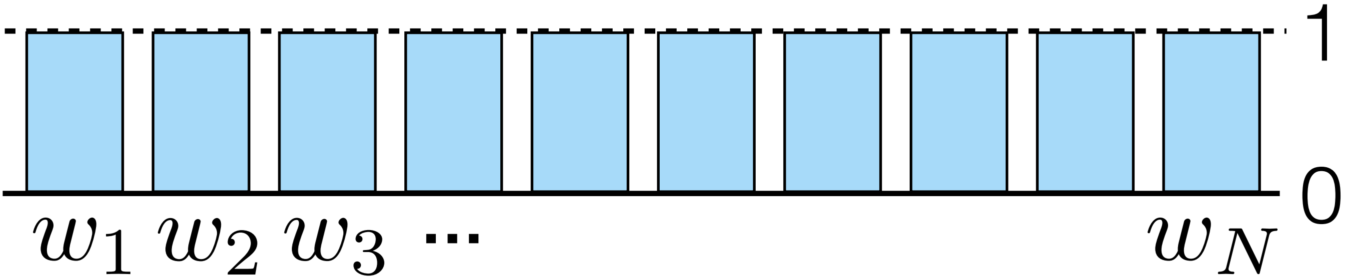

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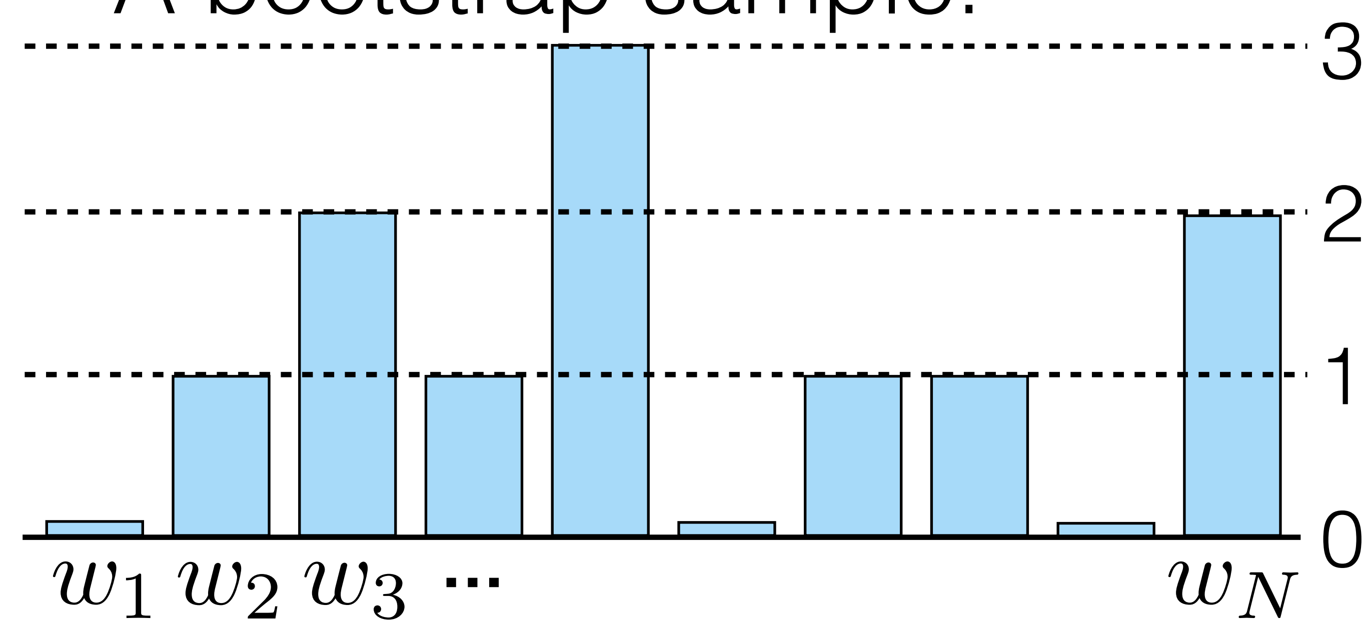

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[Gawron et al 03; Gelman, Hill 07 Sec 13.5]

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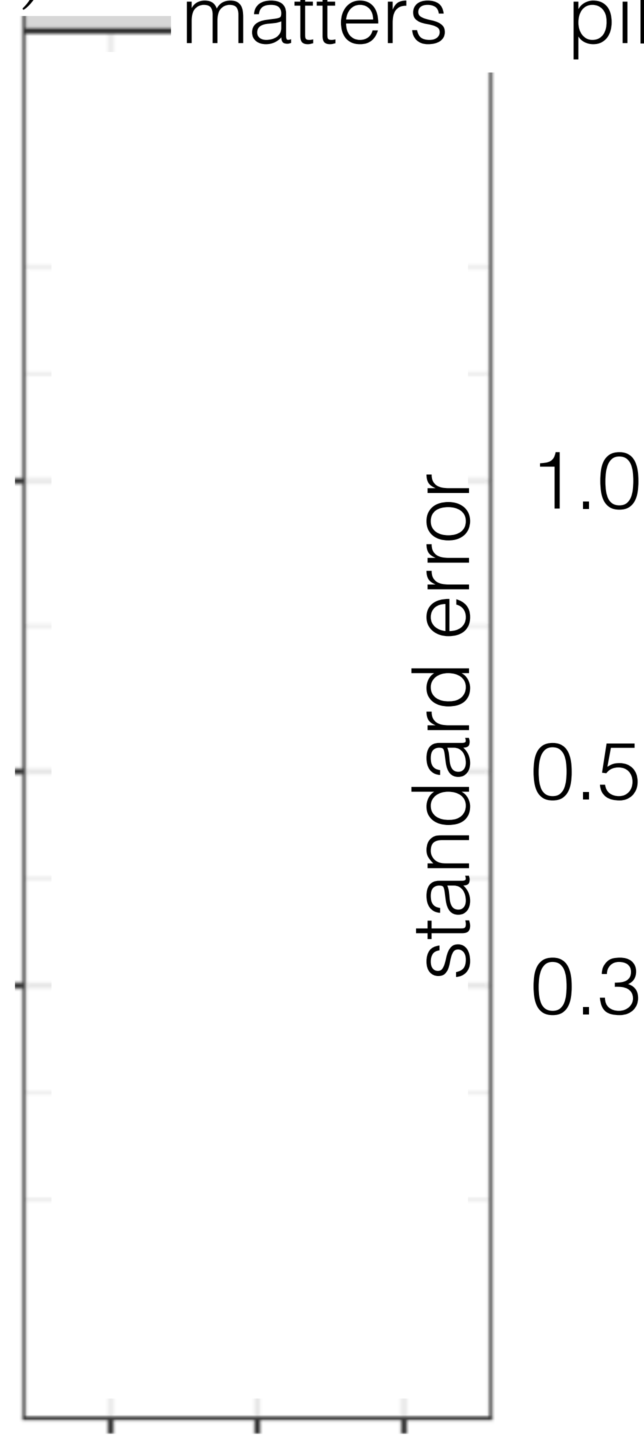

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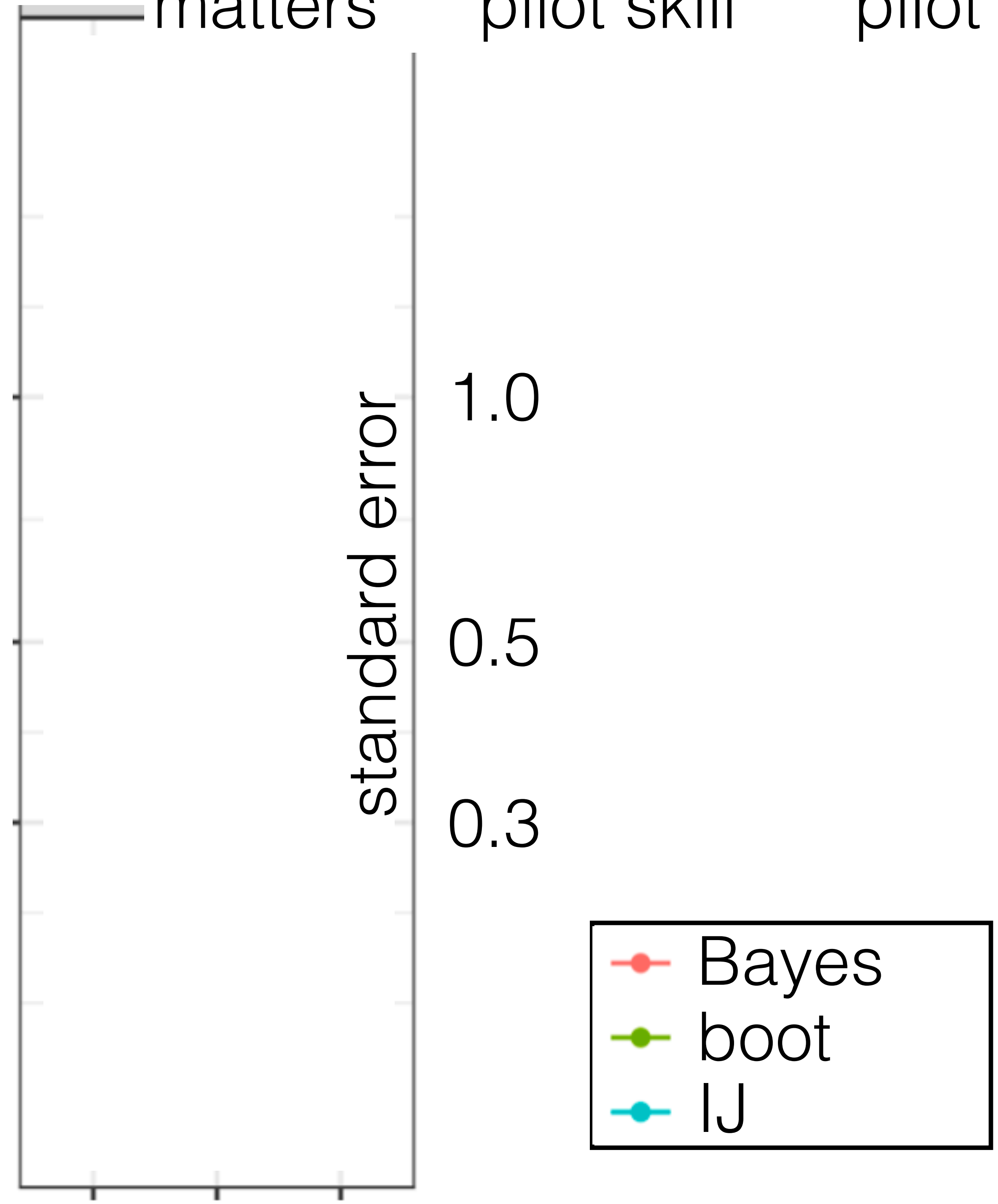

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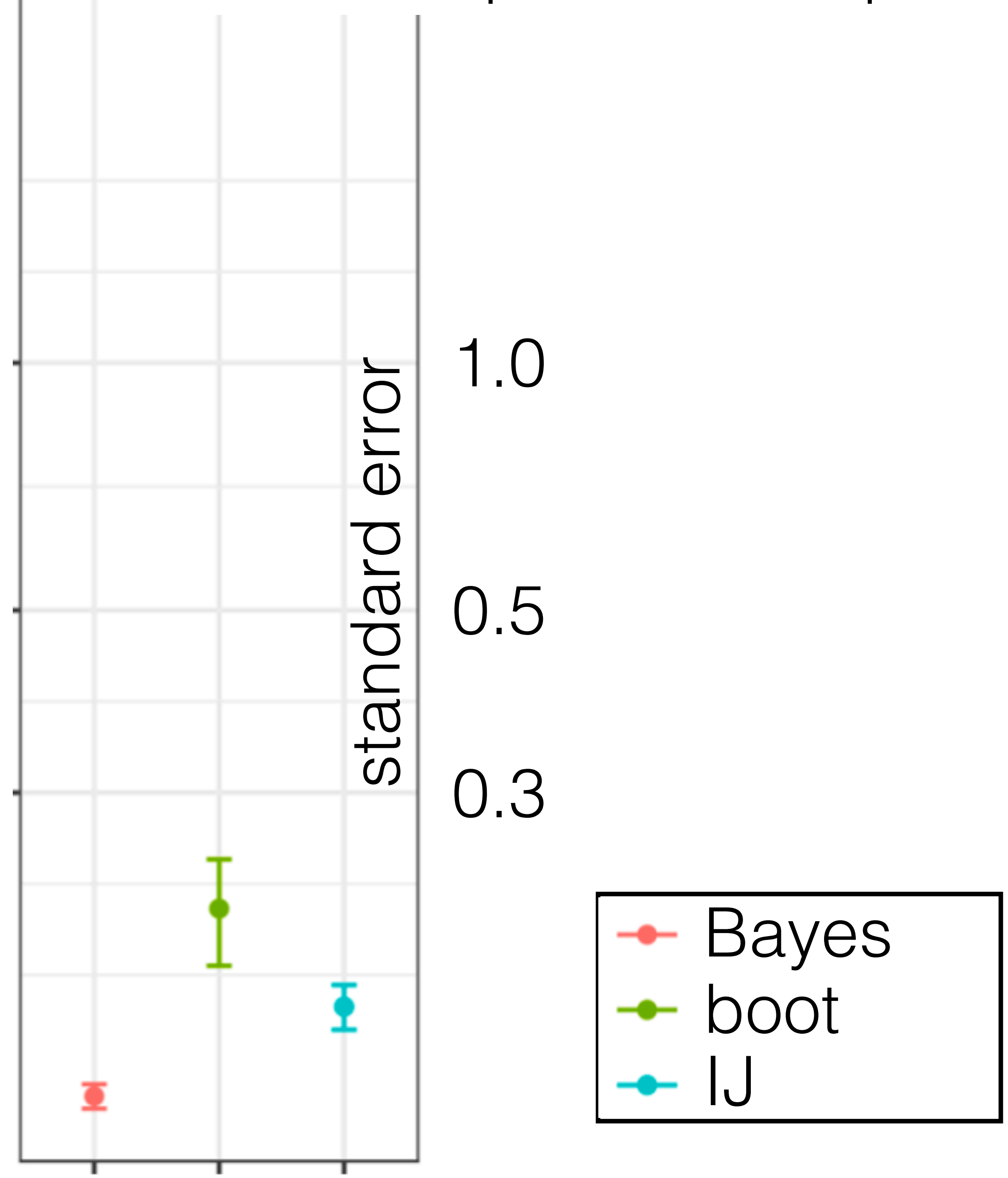

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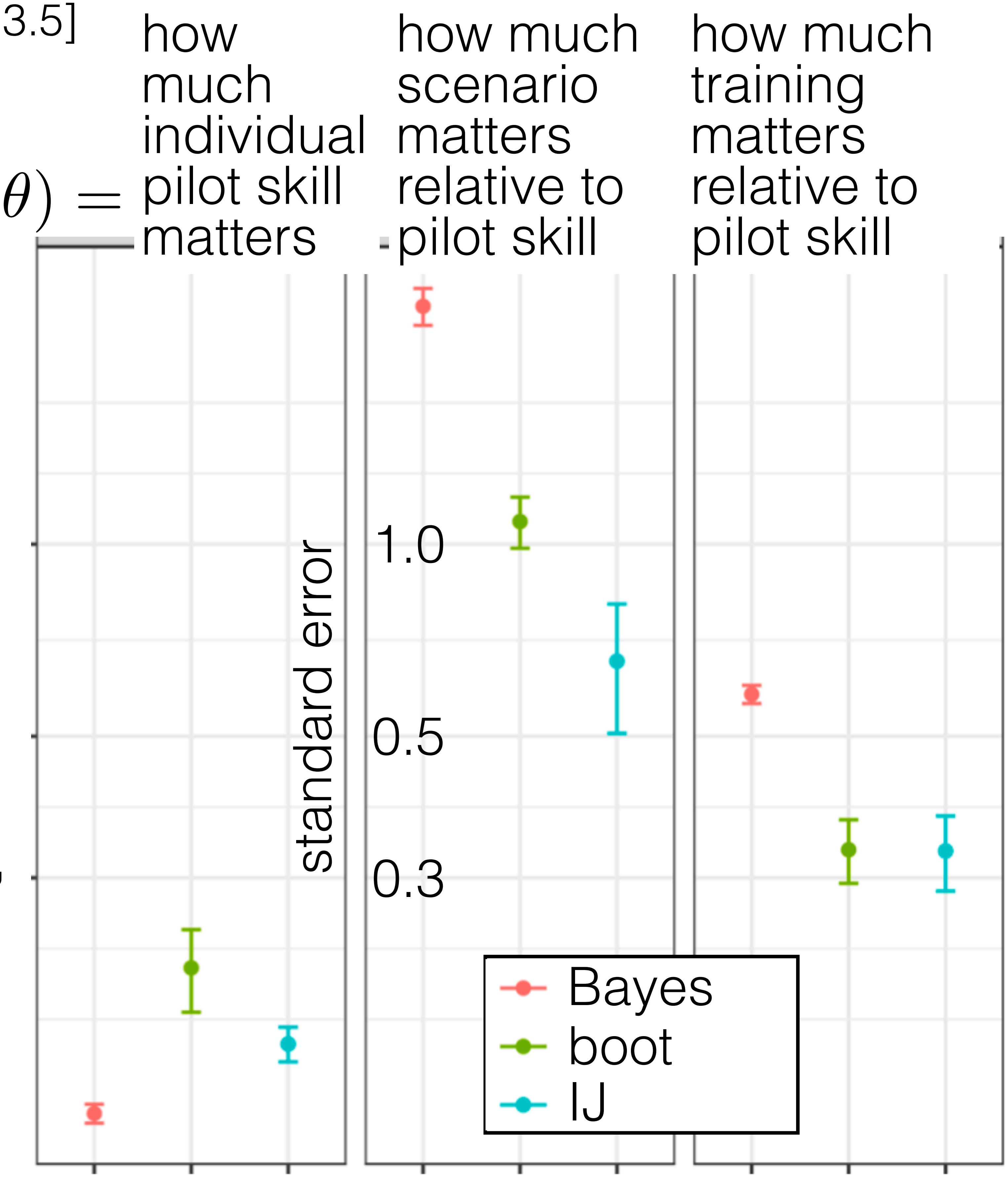

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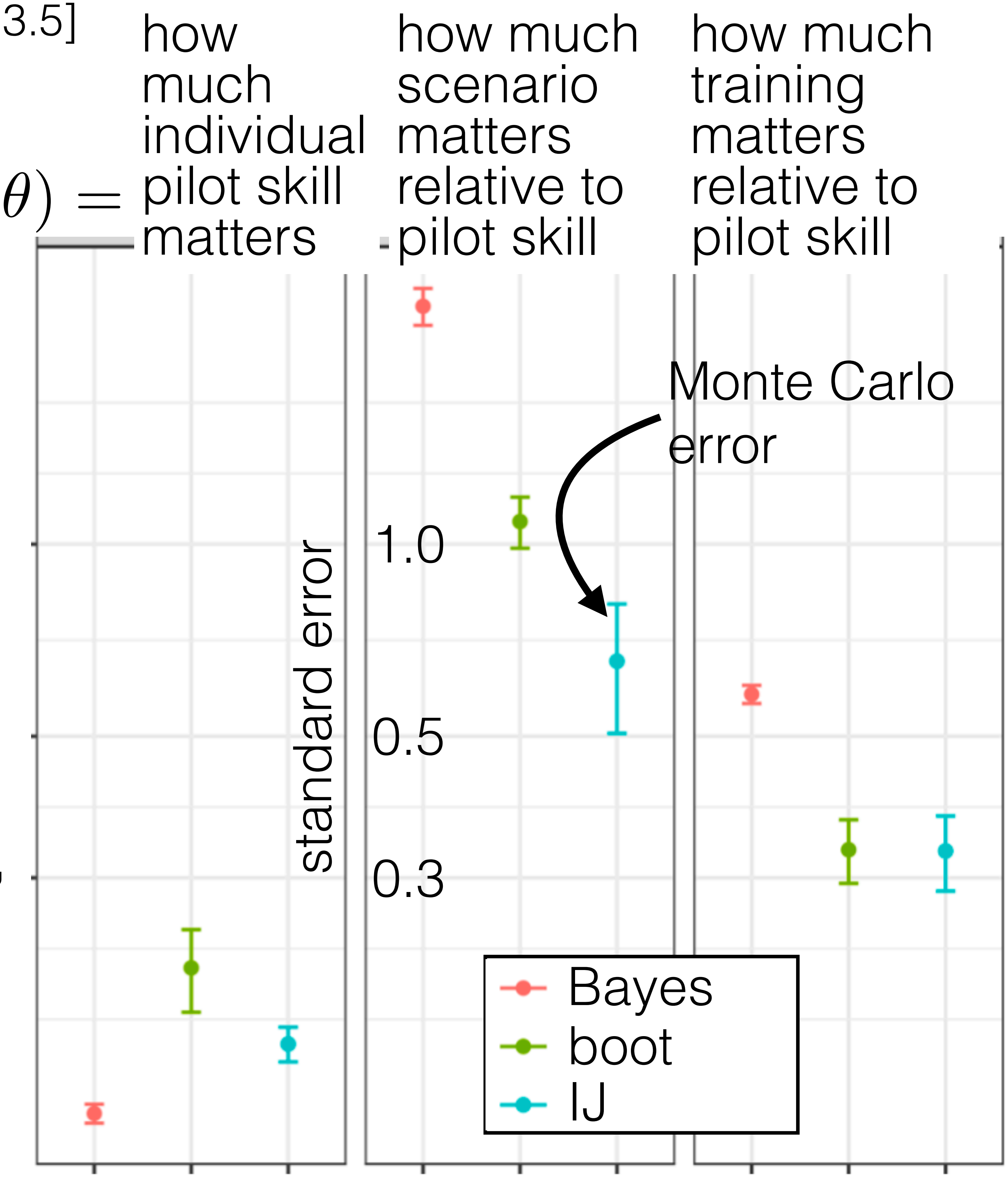

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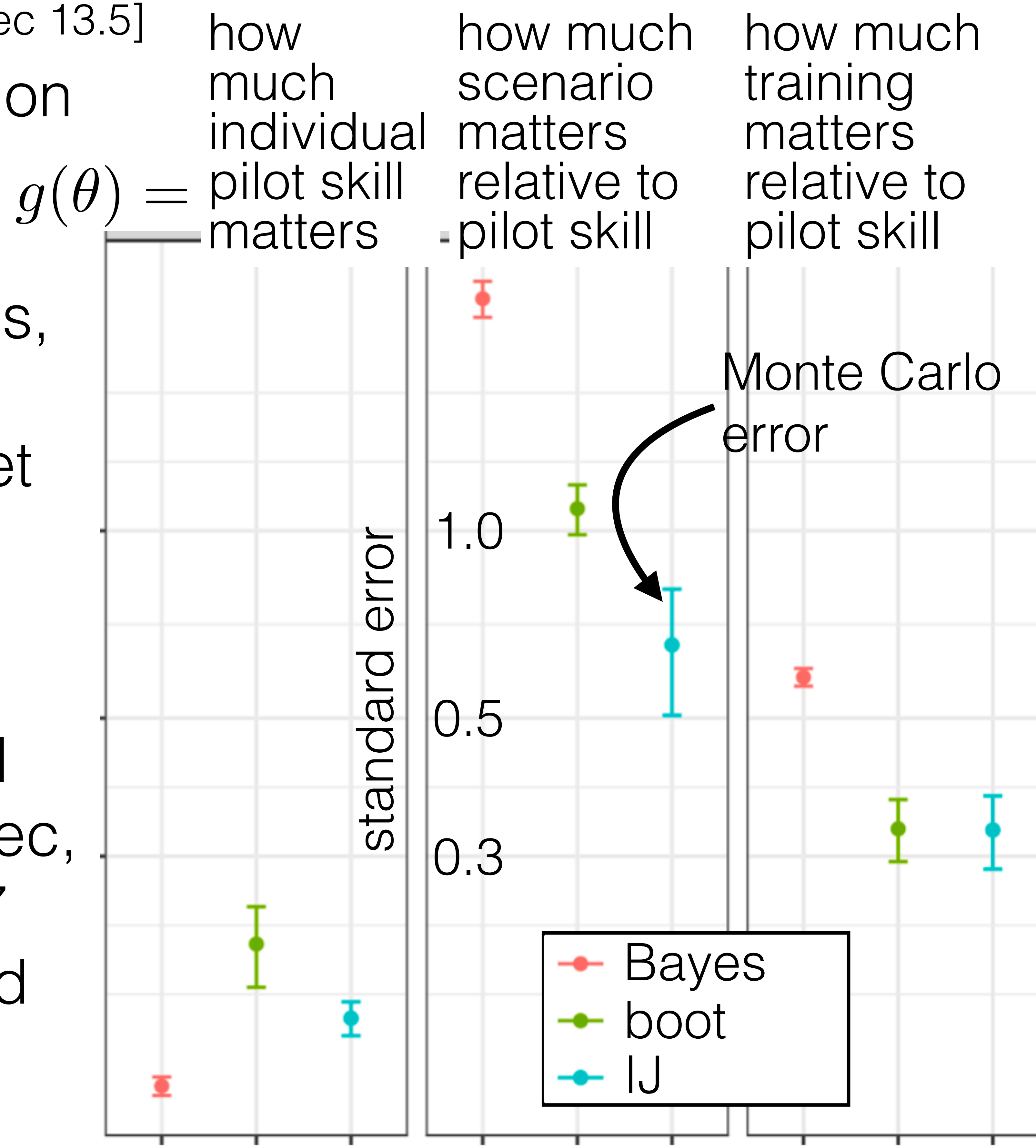

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- **First, take *N* growing and parameter dimension fixed.**
- Take regularity assumptions & ignore posterior approx.
- Let V be such that: $N \underset{\mathbb{F}(X)}{\mathrm{Var}} \underset{p(\theta|X)}{\mathbb{E}} [g(\theta)] \to V$
- Classic Bayesian Central Limit Theorem: $N \underset{p(\theta|X)}{\mathrm{Var}} [g(\theta)] \to U$
  - $U = V$ under correct specification, else generally different
- Huggins and Miller 2019 show bootstrap estimator $\to V$
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- We propose and analyze using the infinitesimal jackknife to estimate frequentist sampling uncertainty of a Bayesian estimator.
- We support our method's accuracy, speed, & automation (+ limitations) with simulated & real experiments and theory
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  - Broderick, Giordano, Meager. An Automatic Finite-Sample Robustness Metric: When Can Dropping a Little Data Make a Big Difference? arxiv.org/abs/2011.14999 (alphabetical)