Simulation-based Inference in Stellar Magnetic Field Models

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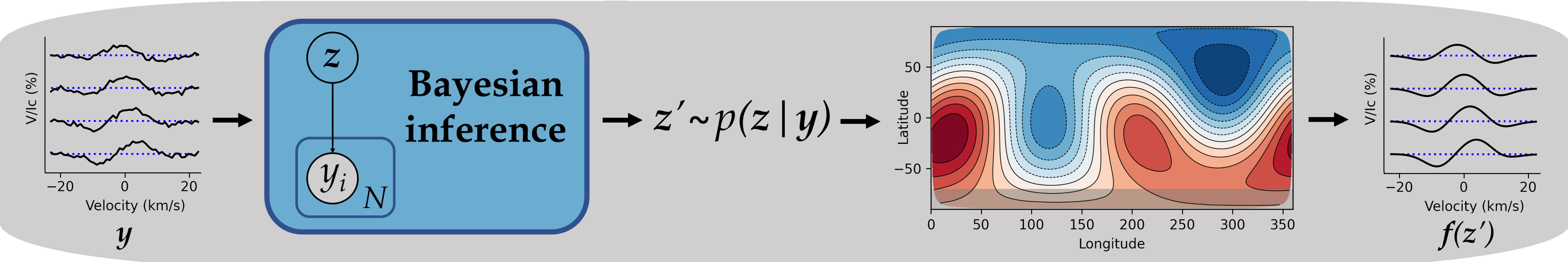
Introduction

- Magnetic fields of stars play a key role in stellar evolutionary processes. Using spectropolarimetric time series observations, the surface magnetic field maps of stars can be reconstructed.
- A major shortcoming of standard inversion techniques is that they only allow for point estimates of the magnetic field maps.
- We propose a probabilistic modeling approach to finding the inverse mapping between two-dimensional magnetic field maps and the observed circular polarization profiles in a fully Bayesian setting. The goal is to quantify the family of field topologies fitting a given set of observations.
- We use a high-dimensional spherical-harmonic magnetic field parameterization, resulting in hundreds of free parameters truncated from an infinite series expansion.

Summary

- We formulate a statistical model capturing the regularization structure favored by standard inversion techniques.
- Hierarchical model extended with priors on stellar parameters.
- MCMC inference using MALA provides reliable uncertainty quantification of stellar magnetic field maps where current research is dominated by point estimates.
- Efficient implementation in JAX: XLA compilation and automatic differentiation through physics simulator.
- Fast and stable convergence despite high dimensionality and complex forward simulation process.
- Demonstration of proposed scheme on the star Tau Scorpii.



extended abstract, code and data will be available at

Simulator

- Local line profiles modeled by the UR analytical solution of the polarized radiative transfer equation.
- Complex simulation process involving weighted integration of local line profiles across discretized surface grid.

Model

Standard Inversion

$$\mathbf{z} = \arg\min_{\mathbf{z}} \|\Lambda^{\frac{1}{2}}(\mathbf{y} - \mathbf{f}(\mathbf{z}))\|_2^2 + r(\mathbf{z})$$

$$r(\mathbf{z}) = \eta \sum_{l=1}^{l_{\max}} \sum_{m=-l}^{l} l^2(\alpha_{l,m}^2 + \beta_{l,m}^2 + \gamma_{l,m}^2)$$

Simplest Statistical Model

$$p(\mathbf{z}|\mathbf{y}) = p(\mathbf{y}|\mathbf{z})p(\mathbf{z})/p(\mathbf{y})$$

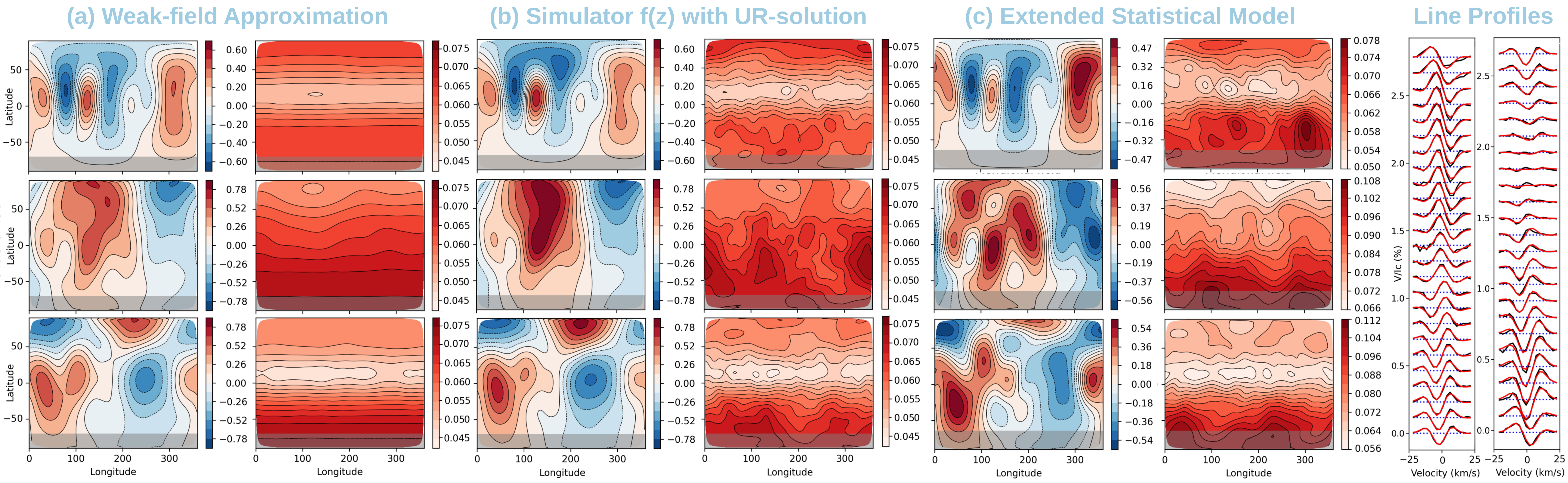
$$p(\mathbf{y}|\mathbf{z}) = \mathcal{N}(\mathbf{y}; \mathbf{f}(\mathbf{z}), \Lambda^{-1})$$

$$p(\mathbf{z}) = \mathcal{N}(\mathbf{0}, \omega^{-1} I), \quad \omega_i = \eta l_i^2$$

Results

- Figure 1 shows the reconstructed field maps and standard deviation maps for bright, massive star Tau Scorpii.
- We show that fast and stable convergence is possible using MCMC methods in a high-dimensional setting with 362 target variables.
- Our JAX implementation achieves a GPU runtime speedup of 648x compared to a corresponding implementation in NumPy.
- Our results formalize empirical uncertainty quantification.
- The approach will be applied to particularly challenging targets with uninformative point estimates, such as stars in eclipsing binary systems.

Figure 1: MAP surface magnetic field maps (kG) and corresponding standard deviation maps for Tau Scorpii using three different simulators: (a) weak-field approximation of simulator f(z), (b) f(z) and (c) weak-field approximation with statistical model extended with priors on projected rotational velocity and stellar inclination.



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